

Project 1: Thermodynamic Cycle Analysis

Mech 477 Aerospace Propulsion



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Submitted To: Dr. Kendal Bushe

Date: October 11, 2026

Abstract

This report presents a zero-dimensional thermodynamic cycle analysis of the Pratt & Whitney F100-PW-100 low-bypass-ratio afterburning turbofan, the original powerplant for the F-15 Eagle air superiority fighter. The engine, characterized by a bypass ratio of 0.69, an overall pressure ratio of 24.1, and a turbine inlet temperature of 1680 K, is modeled using a station-by-station MATLAB solver that enforces mass, momentum, and energy conservation with temperature-dependent gas properties and polytropic efficiency formulations for multistage compression and expansion. The cycle model includes separate core and bypass streams, complete adiabatic mixing, and a fixed-geometry convergent nozzle with explicit choking and mass-continuity constraints.

At the on-design static operating condition (ambient pressure 90.31 kPa, temperature 288.15 K, and total mass flow rate 101.6 kg/s), the analysis predicts a net thrust of 67.52 kN at military power (max power, afterburner off) and 100.53 kN with maximum afterburning, corresponding to deviations of +3.2% and -5.7% from published ratings of 65.3 kN and 106.4 kN, respectively. These discrepancies are primarily attributed to the omission of turbine cooling bleed flows and the idealization of the mixer as a lossless momentum interface. The calculated specific thrust increases from 663.04 N · s/kg in dry operation to 987.37 N · s/kg with afterburning, while the associated specific fuel consumption rises from 0.0711 kg/(N · hr) to 0.1555 kg/(N · hr). Thermodynamically, these values indicate that engaging the afterburner increases net thrust by approximately $1.49\times$ at the cost of more than doubling the fuel required per unit thrust (a 120% increase), highlighting the inherently low propulsive efficiency of reheated operation. A parametric sweep of compressor pressure ratio further demonstrates that the F100's design point ($OPR = 24.1$) favors high specific thrust over minimum SFC, consistent with the mission requirements of an air superiority fighter optimized for combat maneuvering rather than long-range cruise.

In addition to the primary 0D cycle model, a separate Python-based workflow coupling Cantera and CoolProp with real-fuel property data for Jet A-1 was used as an auxiliary check and to explore alternative thermodynamic assumptions; this higher-fidelity framework, which lies outside the main scope of the project, is documented in the appendices as a foundation for more rigorous model development.

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Nomenclature

Flow and thermodynamic quantities

Symbol	Units	Definition
T	K	Static temperature
T_0	K	Stagnation temperature
P	Pa	Static pressure
P_0	Pa	Stagnation pressure
M	–	Mach number
ρ	kg m^{-3}	Density
R	$\text{J kg}^{-1} \text{K}^{-1}$	Specific gas constant
c_p	$\text{J kg}^{-1} \text{K}^{-1}$	Specific heat at constant pressure
γ	–	Ratio of specific heats, $\gamma = c_p/c_v$
h	J kg^{-1}	Static enthalpy
h_0	J kg^{-1}	Stagnation enthalpy
$\dot{m}$	kg s^{-1}	Mass flow rate
$\dot{m}_{\text{air}}$	kg s^{-1}	Total engine air mass flow rate
$\dot{m}_{\text{core}}$	kg s^{-1}	Core-stream air mass flow rate
$\dot{m}_{\text{byp}}$	kg s^{-1}	Bypass-stream air mass flow rate
f	–	Fuel-to-air mass ratio (main combustor)
f_{AB}	–	Additional fuel-to-air ratio (afterburner)
AFR	–	Air-fuel ratio (mass basis)
AFR_{st}	–	Stoichiometric air-fuel ratio
AFR_{act}	–	Actual operating air-fuel ratio
ϕ	–	Equivalence ratio, $\phi = \text{AFR}_{\text{st}}/\text{AFR}_{\text{act}}$
U	m s^{-1}	Axial flow velocity
U_{cmb}	m s^{-1}	Gas velocity in combustor
Q_{LHV}	J kg^{-1}	Fuel lower heating value (Jet A-1)

Cycle and component parameters

Symbol	Units	Definition
α	–	Bypass ratio, $\alpha = \dot{m}_{\text{byp}}/\dot{m}_{\text{core}}$
BPR	–	Bypass ratio
PR_c	–	Overall compressor pressure ratio
π	–	Generic total-pressure ratio, $P_{0,\text{out}}/P_{0,\text{in}}$
π_d	–	Inlet total-pressure recovery (diffuser)
π_f	–	Fan total-pressure ratio
π_c	–	Core compressor total-pressure ratio
π_b	–	Burner total-pressure ratio
π_{AB}	–	Afterburner total-pressure ratio
$\pi_{u,C}$	–	Core-duct total-pressure ratio
$\pi_{u,B}$	–	Bypass-duct total-pressure ratio
π_m	–	Bypass mixer total-pressure ratio
π_e	–	Exhaust pressure ratio, P_e/P_a
TIT	K	Turbine inlet stagnation temperature
$T_{0,\text{AB}}$	K	Afterburner exit stagnation temperature
A_e	m ²	Nozzle exit area

Efficiencies and performance metrics

Symbol	Units	Definition
$\eta_{p,f}$	–	Polytropic efficiency of the fan
$\eta_{p,c}$	–	Polytropic efficiency of the core compressor
$\eta_{p,t}$	–	Polytropic efficiency of the turbine
η_b	–	Main combustor efficiency
η_{AB}	–	Afterburner combustion efficiency
η_m	–	Mechanical (shaft) efficiency
η_n	–	Nozzle efficiency
$\dot{W}$	W	Shaft power
$\dot{W}_f$	W	Fan power requirement
$\dot{W}_c$	W	Core compressor power requirement
$\dot{W}_t$	W	Turbine power output
F	N	Net thrust
τ	N s kg ⁻¹	Specific thrust, $\tau = F/\dot{m}_{\text{air}}$
U_e	m s ⁻¹	Nozzle exit velocity
SFC	kg N ⁻¹ h ⁻¹	Specific fuel consumption, $\dot{m}_f/F$
η_{th}	–	Thermal efficiency
η_p	–	Propulsive efficiency
η_0	–	Overall efficiency, $\eta_0 = \eta_{\text{th}}\eta_p$

1 Introduction

1.1 Motivation for Zero-dimensional Cycle Analysis

Zero-dimensional (0D) thermodynamic cycle models sit at the bottom of the gas-turbine fidelity ladder, but they carry most of the intuition. Each major component of the engine is reduced to a steady control volume with spatially averaged properties and a small set of performance parameters: total pressure ratios, polytropic efficiencies, pressure losses and shaft efficiency [7, 8, 9]. This is a crude approximation compared to Chemical Reactor Networks (CRN), 2D CFD or meanline solvers, but it is fast, transparent, and easy to extend. Thermodynamic engine modelling has two fundamental components that must be characterized: the total system thermodynamics and the component-level energy and flow equations. In a thermodynamic model, system modeling is centered around the Brayton cycle, where relationships between pressure, temperature, entropy, and enthalpy can be developed [9].

For a mixed-flow, low-bypass-ratio afterburning turbofan, a good 0D model is enough to answer the basic questions: How much thrust is produced in dry and afterburning operation? What is the fuel-flow penalty for lighting the afterburner? How sensitive are thrust and specific fuel consumption to compressor pressure ratio, turbine inlet temperature and component losses? A 0D framework built carefully with non-ideal components and temperature-dependent properties can hit “engineering accuracy” on these questions while staying simple enough to live in a batched script.

The present work uses the Pratt & Whitney F100-PW-100 as a case study. The objective is to construct a rigorous 0D model of this engine at an on-design sea-level static (SLS)¹ operating point, and to quantify how far such a model can go before its assumptions start to bite.

1.2 F100-PW-100 Engine Characteristics

The Pratt & Whitney F100-PW-100 is an afterburning, low-bypass-ratio turbofan developed in the early 1970s for the F-15 Eagle air-superiority fighter [5, 4], as illustrated in Fig. 1.1. Key geometric, component and performance data drawn from museum records and open literature are summarised in Table 1.1. At sea-level static conditions the engine is rated at

¹Sea-level static (SLS), refers to a ground-test condition with negligible flight Mach number and reference ambient conditions. For this project, the on-design point is defined by $P_a = 90.31$ kPa and $T_a = 288.15$ K, consistent with the project specification and F100 static test data [10, 3].

approximately 55.2 kN of continuous dry thrust, 65.3 kN at *military power*², and 106.4 kN with full afterburner, with an overall pressure ratio of about 24:1 and an air mass flow rate close to 102 kg/s [3, 12]. These published figures provide the reference against which the predictions of the 0D cycle model are compared.

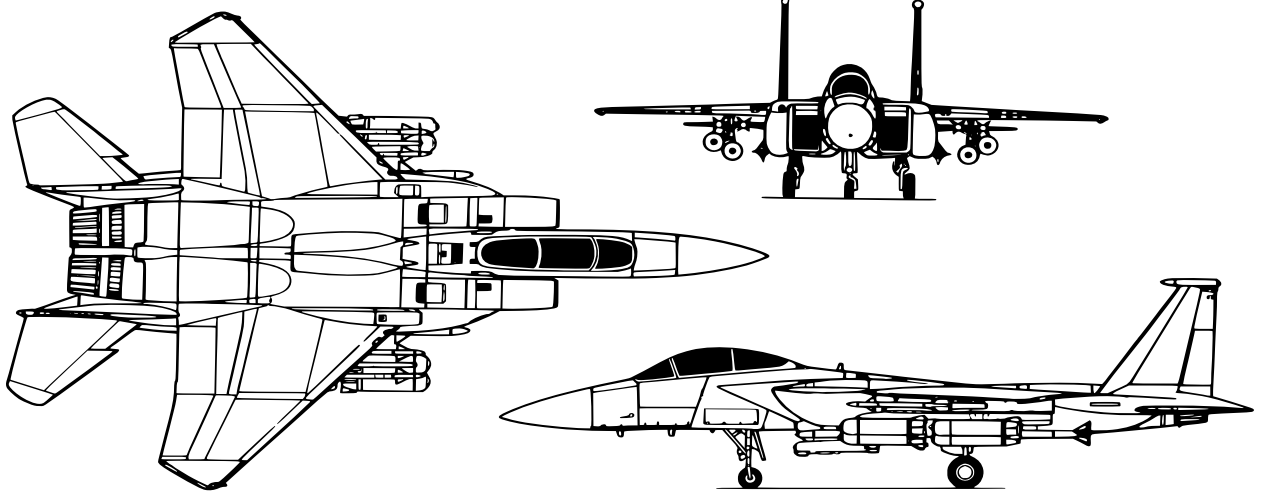


Figure 1.1: Schematic of the F-15E Strike Eagle. Image modified from [1].

²In fighter-engine terminology, *military power* denotes the maximum rated thrust with afterburner off, typically time-limited but intended for combat manoeuvres [11, 7].

Table 1.1: Published specifications of the Pratt & Whitney F100–PW–100 engine, based on museum data and open literature sources [3, 4, 5].

Quantity	Value (metric)
Engine type	Afterburning turbofan
Length	4.85 m
Inlet diameter	0.88 m
Maximum external diameter	1.18 m
Dry weight	1377 kg
Compressor	Dual-spool axial; 3-stage fan, 10-stage compressor
Combustor	Annular
Turbine	2-stage high-pressure, 2-stage low-pressure
Dry thrust (continuous)	55.2 kN
Military power thrust (Max no AB)	65.3 kN
Maximum thrust with full afterburner	106.4 kN
Overall pressure ratio	24:1
Air mass flow rate	102 kg/s
Thrust-to-weight ratio	7.88:1

Architecturally, the F100 is a two-spool mixed-flow turbofan: fan and core streams are recombined upstream of the afterburner, and the entire mass flow is reheated and expanded through a single convergent–divergent nozzle. The mixing process introduces a modest total-pressure loss in dry operation but allows the afterburner to act on the full mass flow when lit, which is precisely the trade a fighter designer makes when exchanging fuel for thrust [13, 2]. The generic layout and station numbering adopted in this work are shown in Fig. 1.2.

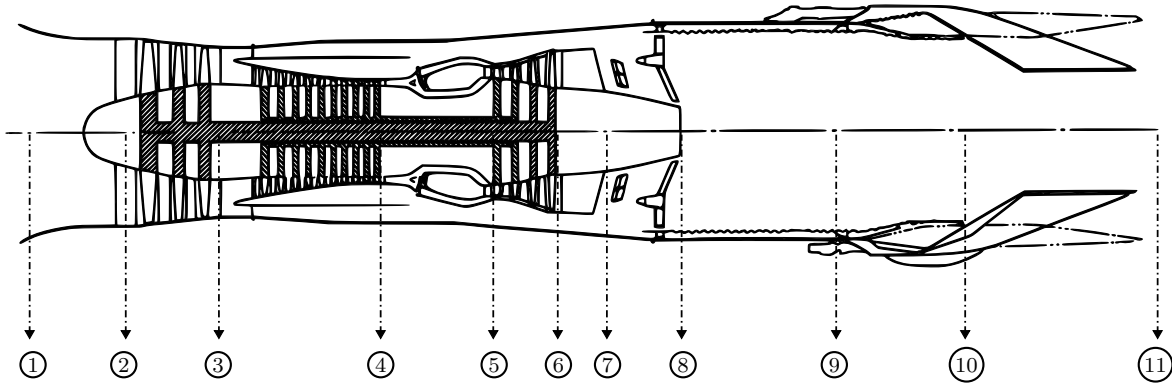


Figure 1.2: Schematic of the F100-PW-100 mixed-flow afterburning turbofan, showing inlet, fan, core compressor, combustor, turbines, mixer, afterburner and convergent nozzle with station numbering, adapted from [2].

In this report the F100-PW-100 thermodynamic cycle is described using the station numbering in Fig. 1.2. Ambient air upstream of the airframe is denoted Station ①, and is decelerated and compressed in the inlet-diffuser to Station ②. The fan raises the total pressure between Stations ② and ③, where the flow is split into core and bypass streams. The core stream is further compressed by the core compressor from Station ③ to Station ④, then heated in the main combustor between Stations ④ and ⑤. Turbine stages extract shaft work between Stations ⑤ and ⑥ to drive the fan and compressor. Downstream of the turbine, the hot core stream at Station ⑥ meets the bypass stream at Station ⑦; the two streams mix in the mixer control volume denoted Station ⑧. When the afterburner is lit, additional fuel is injected and burned between Stations ⑧ and ⑨. The mixed, (optionally) reheated flow then expands through the convergent nozzle from Station ⑩ to the exhaust plane at Station ⑪ (also denoted Station e), where the exit momentum and pressure difference relative to the ambient generate thrust.

1.3 On-design condition and given specifications

The assignment defines a single on-design operating point: the engine mounted on a fixed test bench at SLS, with prescribed component performance parameters. The set of inputs for the 0D model is summarised in Table 1.2, exactly as given in the project statement.

Table 1.2: Specifications of the F100-PW-100 engine, reproduced from project outline [6].

Air Flow Rate [kg/s]	101.6	Intake Pressure Ratio	0.99
Fan Pressure Ratio	3.00	Compressor Pressure Ratio	8.04
Polytropic Efficiency of the Compressor	0.83	Polytropic Efficiency of the Fan	0.79
Compressor Duct Pressure Ratio	0.99	Bypass Ratio	0.69
Bypass Duct Pressure Ratio	0.93	Fuel Heating Value [kJ/kg]	43323.0
Burner Design Efficiency	0.98	Burner Pressure Ratio	0.96
Mechanical Efficiency	0.99	Polytropic Efficiency of the Turbine	0.86
Turbine Inlet Temperature [K]	1680	Mixer Pressure Ratio	0.99
Nozzle Efficiency	0.99	Temp. of the Afterburned Gas [K]	2035.4

The on-design condition of this engine is considered to be on the ground with the ambient pressure of 90.31 kPa and the ambient temperature of 288.15 K. At this on-design point the fan and compressor operate at fixed total-pressure ratios, the burner is sized to reach a turbine inlet temperature of 1680 K in dry operation, and the afterburner when lit, is specified to raise the mixed-stream stagnation temperature to $T_{0,AB} = 2035.4$ K. A fixed-area convergent nozzle with exit area $A_e = 0.726$ m² is assumed in the project statement, and the mixer is treated as perfectly mixed with a small, prescribed total-pressure loss.³ We are to assume that the gas mach number at the leading edge of the fan blades and the turbine exit plane is 0.5. Furthermore, assume a constant velocity of 150 m/s for the gas flow in the combustion chamber.

1.4 Interpretation of the given parameters for the 0D model

The parameters supplied in the project specification are not just numerical inputs; they define the thermodynamic degrees of freedom that the zero-dimensional cycle model is allowed to manipulate. A 0D solver has no spatial resolution, so every quantity must be interpreted as a station-averaged property, and every component parameter establishes either a constraint or a closure relation for the underlying conservation equations.

- **Air mass flow rate and bypass ratio** determine the physical scale of the engine and the core-to-bypass mass partitioning. For the F100-PW-100, the specified $\dot{m}_{\text{air}}$ and $\alpha = 0.69$ lock the solver into the low-bypass, high-specific-thrust regime typical of fourth-generation fighter propulsion.

³The modelling assumptions on inlet pressure recovery, duct losses, mixer behaviour, and nozzle area follow the project handout [6].

- **Pressure ratios across the inlet, ducts, burner, mixer and afterburner** describe how total pressure is accumulated through compression and dissipated through mixing, combustion and frictional losses. These parameters directly control the total-pressure budget and therefore the amount of expansion work that can be extracted in the turbines or converted into jet kinetic energy.
- **Fan, compressor and turbine polytropic efficiencies** encode the multi-stage nature of turbomachinery. Polytropic efficiency is the correct quantity to use in cycle analysis because it provides a pressure-ratio-independent measure of incremental stage performance [14]. These efficiencies directly control the temperature rise or drop required to achieve a prescribed pressure ratio.
- **Combustion-related parameters** (burner efficiency, heating value, turbine inlet temperature, afterburner exit temperature) establish the chemical energy addition and the thermal loading limits of the hot section. The specified TIT and afterburner temperature effectively bound the achievable thrust in dry and augmented operation.
- **Mechanical and nozzle efficiencies** determine how much turbine power can actually be delivered to the compressor and fan, and how much of the remaining enthalpy can be converted into jet velocity. These parameters close the energy balance on the spools and the momentum balance at the nozzle.

A key complication is the strong temperature dependence of $c_p(T)$ and $\gamma(T)$ over the 300–2200 K range spanned by the F100. Component exit temperatures are not known in advance; they are found by enforcing energy balance with temperature-dependent thermodynamic properties [9, 8]. This creates circular dependencies: the temperature rise required for a given pressure ratio depends on $c_p(T)$, but c_p itself depends on the (unknown) exit temperature. These dependencies must be resolved iteratively, and neglecting them produces non-negligible errors in thrust and SFC for high-OPR engines [14, 15, 16].

1.5 Scope and limitations of the present analysis

The present study intentionally focuses on a constrained problem so that the physics of the F100–PW–100 cycle can be examined cleanly.

- **Only the on-design, sea-level static operating point is modeled.** No off-design matching, variable geometry, corrected-speed scaling or mission analysis is attempted. This isolates the fundamental cycle behaviour from operational effects.
- **Every component is treated as a 0D steady control volume with uniform station properties.** Radial gradients, blade-row aerodynamics, leakage flows and unsteady phenomena (stall, surge, instabilities) are outside the scope of a 0D solver.

- **Combustion is not explicitly modeled.** The combustor and afterburner are implemented as heat-addition devices that drive the flow to specified stagnation temperatures. The required fuel flow is determined from an enthalpy balance rather than from chemical kinetics. No species modeling, dissociation, or emissions prediction is attempted in this script.
- **Mixer loss and nozzle geometry are treated as fixed inputs.** The model accepts these as design parameters from the specification and performs no optimization of mixing area, nozzle throat size or pressure-ratio scheduling.

Within these limits, the 0D solver predicts net thrust, specific thrust, and TSFC for military-power (dry) and maximum-afterburner operation and evaluates their sensitivity to airflow, compressor pressure ratio, TIT, and component polytropic efficiencies. The detailed thermodynamic rationale for these assumptions is expanded in the Methodology section; here they define the operating boundaries of the cycle model.

2 Methodology

The analysis employs a steady, zero-dimensional thermodynamic cycle model that tracks stagnation temperature T_0 and stagnation pressure P_0 at each station listed in Fig. 1.2. The objective is to reproduce the on-design, sea-level static (SLS) performance of the F100-PW-100 using the parameters in Table 1.2. The model directly enforces conservation of mass, momentum and energy across each component, with temperature-dependent thermodynamic properties and polytropic compression and expansion models forming the backbone of the calculation.

2.1 Assumptions

The zero-dimensional cycle model is built on a set of physically motivated, computationally tractable assumptions. These assumptions follow standard practice in preliminary gas turbine performance analysis [7, 8] and are consistent with the thermodynamic framework and nomenclature established earlier.

A1: Steady, Quasi-One-Dimensional Flow All components are treated as steady control volumes. Flow properties at each station are uniform across the annulus and constant in time. Radial gradients, swirl, unsteadiness, shock oscillation, stall, and surge phenomena are not modeled.

A2: Ideal Gas with Temperature-Dependent Properties The working fluid (air)

obeys the ideal gas law, but thermodynamic properties vary with temperature:

$$c_p = c_p(T), \quad \gamma = \gamma(T), \quad h = h(T).$$

This is essential for the F100, where temperatures span roughly 290–2200 K. Constant property assumptions produce systematic thrust and SFC errors in high-OPR engines [9, 14].

- A3: Polytropic Efficiencies for Turbomachinery** The fan, compressor, and turbine are modeled using their respective polytropic efficiencies $\eta_{p,f}$, $\eta_{p,c}$ and $\eta_{p,t}$. These parameters represent infinitesimal stage performance and remain nearly independent of pressure ratio, unlike isentropic efficiencies.
- A4: Specified Total Pressure Losses** Losses in the intake, bypass duct, compressor duct, burner, mixer, and afterburner are represented using fixed total pressure ratios:

$$\pi_d, \pi_{u,C}, \pi_{u,B}, \pi_b, \pi_{AB}, \pi_m.$$

No detailed viscous modeling, boundary layer analysis, bleed extraction, or shock-induced losses are included.

- A5: Combustion Represented Through Enthalpy Addition** Combustion is not modeled through a fuel–air ratio prescription. Instead, fuel flow is obtained from an enthalpy balance using $h(T)$:

$$h_{05} - h_{04} = \eta_b f Q_{LHV}.$$

Chemical kinetics, species dissociation, equilibrium chemistry, and emissions modeling are excluded. These are deferred to future Canterra-based reactive simulations.

- A6: Fluid Composition Held Fixed** The mixture is treated as a single-component calorically variable gas. Molecular weight and species composition remain fixed. Only $c_p(T)$, $\gamma(T)$, and $h(T)$ vary with temperature. No multi-species, equilibrium, or frozen-flow chemistry is included.
- A7: Adiabatic Components (Except Burners)** All components except the main combustor and afterburner are assumed adiabatic. Heat transfer to structure or environment is neglected. Shaft losses are included solely through the mechanical efficiency η_m .
- A8: Bypass–Core Mixing Treated as Idealized Adiabatic Mixing** Mixing occurs at equal static pressure. The mixer exit stagnation temperature is obtained

from an enthalpy balance. The mixer exit stagnation pressure is a mass-weighted average multiplied by the loss factor π_m . No shear layer modeling, turbulence modeling, or diffuser effects are included.

A9: Nozzle Flow Modeled with Choking Logic and Free-Jet Expansion The nozzle uses compressible flow relations with temperature dependent $\gamma(T)$. Choking is detected via the critical pressure ratio. If the fixed geometric area cannot support the mass flow at the given pressure ratio, internal free-jet expansion to ambient is assumed to prevent nonphysical over expansion.

A10: Geometry and Operating Point Fixed The analysis is restricted to the sea-level static (SLS) on-design condition. In “design mode,” the nozzle area is solved to pass the required mass flow. Otherwise, the nozzle area is fixed at the maximum afterburning geometry $A_e = 0.726 \text{ m}^2$. Variable geometry mechanisms, off-design matching, and control system effects are not included.

Additional modelling assumptions that are specific to individual components or numerical procedures are introduced in the corresponding subsections that follow. The list above defines only the global assumptions of the cycle model and is not intended to be exhaustive.

2.2 System architecture

The F100–PW–100 is a two-spool, mixed-flow, low-bypass-ratio turbofan with an annular main combustor and a full-flow afterburner. A block-level view of the engine is shown in Fig. 2.1, highlighting the major components and the way energy moves through the system. Ambient air enters the inlet, is diffused and delivered to the fan, where the flow is split into a core stream and a bypass stream. The core stream passes through a low-pressure compressor (LPC) followed by a high-pressure compressor (HPC), is heated in the main combustor, and then expands through a high-pressure turbine (HPT) and a low-pressure turbine (LPT) before entering the mixer. The bypass stream is routed through the bypass duct and rejoins the hot core stream upstream of the afterburner. Downstream of the mixer, fuel is injected again in the afterburner when lit, and the mixed flow finally expands through the converging nozzle to generate exhaust kinetic energy.

The diagram in Fig. 2.1 therefore represents a more detailed physical architecture than the one used in the cycle model. In the actual engine, the LPC and HPC are separate axial compressor assemblies on the low- and high-pressure spools, and the HPT and LPT are distinct turbine stages driving the high- and low-pressure shafts. In the present zero-dimensional analysis, these subcomponents are deliberately grouped into a single effective compressor and a single effective turbine. Thermodynamically, the cycle “sees” the combined

pressure ratio and aggregate polytropic efficiency of each compression and expansion process; splitting them into LPC/HPC and HPT/LPT does not change the station-level conservation equations used in the model. The block diagram is therefore used to make the real engine architecture explicit, while the mathematical model operates on the reduced representation consisting of:

- a single effective compressor with overall pressure ratio PR_c and polytropic efficiency $\eta_{p,c}$;
- a single effective turbine with overall expansion ratio and polytropic efficiency $\eta_{p,t}$;
- explicit inlet, bypass duct, mixer, main combustor, afterburner and nozzle elements, each with their own loss or efficiency parameters as defined in the nomenclature and Table 1.2.

This abstraction keeps the core/bypass/mixing/afterburning structure correct while avoiding blade-row and spool-matching detail that does not affect the station enthalpy–pressure states used in the 0D cycle solver.

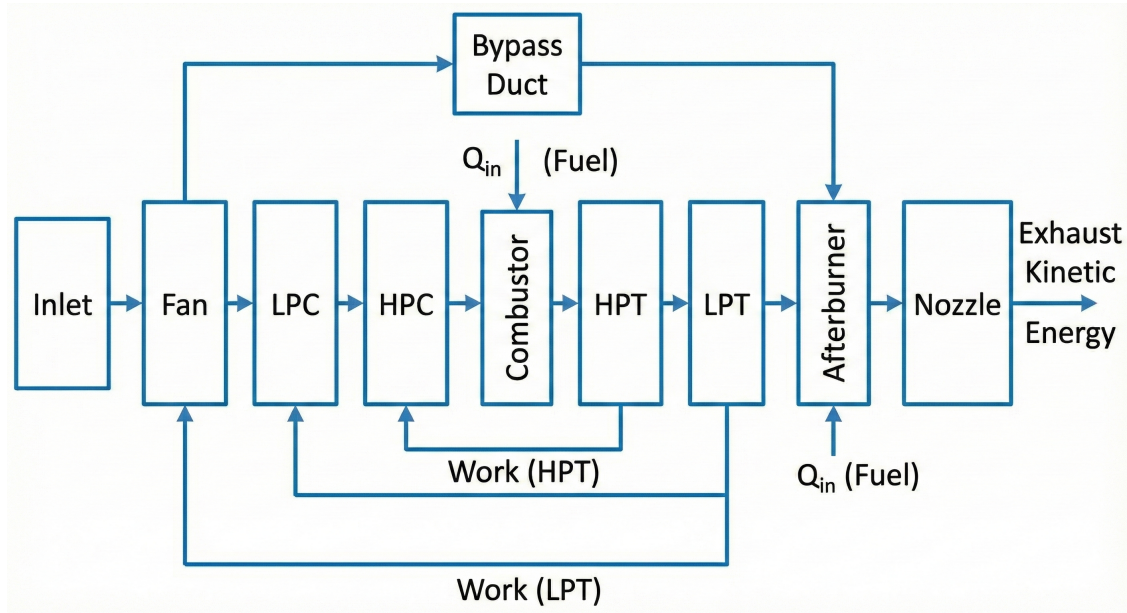


Figure 2.1: Block-diagram representation of the F100–PW–100 engine architecture, showing inlet, fan, low- and high-pressure compressors (LPC, HPC), main combustor, high- and low-pressure turbines (HPT, LPT), bypass duct, afterburner and nozzle, as well as the main work and fuel flow paths. In the present 0D model, (LPC+HPC) and (HPT+LPT) are grouped into single effective compressor and turbine elements.

2.3 Component governing equations

The cycle model advances station by station using stagnation (total) pressure P_0 and temperature T_0 as the primary thermodynamic variables. Static quantities are recovered only where required for nozzle performance and thrust evaluation. Thermodynamic properties are evaluated using temperature-dependent functions for specific heat at constant pressure $c_p(T)$, enthalpy $h(T)$ and standard-state entropy $s^0(T)$, consistent with a semi-perfect gas model [7, 9, 14]. For each station,

$$h(T) = \int_{T_{\text{ref}}}^T c_p(\theta) d\theta, \quad (2.1)$$

$$s(T, P) = s^0(T) - R \ln\left(\frac{P}{P_a}\right), \quad (2.2)$$

$$\gamma(T) = \frac{c_p(T)}{c_p(T) - R}, \quad (2.3)$$

where R is the gas constant and P_a is the ambient static pressure. The MATLAB functions `cp_poly`, `h_poly` and `s_poly` implement Eqs. (2.1) and (2.2) using 4th-order polynomials.

Throughout this section, quantities that are not known a priori and must be determined numerically in the solver are highlighted in **red**. These are the temperatures, pressures or areas that are implicitly defined by the governing equations rather than directly specified as inputs.

Intake: Station ① to Station ②

At sea-level static (SLS) conditions the free stream is assumed to be at rest, so the ambient static and total conditions are identical:

$$T_a = T_{0,a} = 288.15 \text{ K}, \quad P_a = P_{0,a} = 90.31 \text{ kPa}. \quad (2.4)$$

The inlet-diffuser is treated as adiabatic with a prescribed total-pressure recovery factor $\pi_d = \text{PR}_{\text{intake}} < 1$:

$$T_{0,2} = T_{0,a} = T_a, \quad (2.5)$$

$$P_{0,2} = \pi_d P_a. \quad (2.6)$$

In the code these appear as `T02 = Ta` and `P02 = Pa*PR_intake`. No iteration is required in the intake; all quantities are **assumed** from the operating condition and intake pressure ratio.

Fan: Station ② to Station ③

The fan is modelled as a polytropic compressor. The fan total-pressure ratio $\pi_f = \text{PR}_{\text{fan}}$ is specified, and the polytropic efficiency $\eta_{p,f}$ relates the total-temperature ratio to the total-pressure ratio for an ideal gas with mean specific heat $\bar{c}_p$ between Stations ② and ③:

$$\frac{T_{0,3}}{T_{0,2}} = \left(\frac{P_{0,3}}{P_{0,2}} \right)^{\frac{R}{\bar{c}_p \eta_{p,f}}} = \pi_f^{\frac{R}{\bar{c}_p \eta_{p,f}}}. \quad (2.7)$$

The outlet total pressure is

$$P_{0,3} = \pi_f P_{0,2}. \quad (2.8)$$

Because $\bar{c}_p$ depends on the unknown fan outlet temperature, $T_{0,3}$ is obtained via **fixed-point iteration**. The solver repeatedly updates $\bar{c}_p$ using the arithmetic mean of inlet and current outlet temperatures until convergence. The corresponding total enthalpy is $h_{03} = h(T_{0,3})$.

The total engine air mass flow rate $\dot{m}_{\text{air}}$ and bypass ratio α (BPR) determine the core and bypass mass flows:

$$\dot{m}_{\text{core}} = \frac{\dot{m}_{\text{air}}}{1 + \alpha}, \quad \dot{m}_{\text{byp}} = \dot{m}_{\text{air}} - \dot{m}_{\text{core}}. \quad (2.9)$$

Core compressor: Station ③ to Station ④

The effective core compressor combines the physical LPC and HPC into a single polytropic compressor with specified total-pressure ratio $\pi_c = \text{PR}_{\text{comp}}$ and polytropic efficiency $\eta_{p,c}$. A small total-pressure loss in the compressor duct is represented by $\pi_{u,C} = \text{PR}_{\text{duct},C}$:

$$P_{0,2.5} = \pi_{u,C} P_{0,3}, \quad (2.10)$$

$$P_{0,4} = \pi_c P_{0,2.5}. \quad (2.11)$$

The total-temperature rise is obtained from the polytropic relation

$$\frac{T_{0,4}}{T_{0,3}} = \left(\frac{P_{0,4}}{P_{0,3}} \right)^{\frac{R}{\bar{c}_p \eta_{p,c}}} = (\pi_{u,C} \pi_c)^{\frac{R}{\bar{c}_p \eta_{p,c}}}. \quad (2.12)$$

As with the fan, $T_{0,4}$ is solved via fixed-point iteration to account for the temperature dependence of $\bar{c}_p$. The corresponding enthalpies h_{04} and h_{03} are then known.

Main combustor: Station ④ to Station ⑤

The main combustor is modelled as a steady-flow control volume with prescribed burner total-pressure ratio $\pi_b = \text{PR}_{\text{burn}} < 1$ and burner efficiency η_b . The turbine inlet total temperature $T_{0,5}$ (denoted T_{05} in the code) is specified as an input representing material temperature limits:

$$P_{0,5} = \pi_b P_{0,4}, \quad (2.13)$$

$$h_{05} = h(T_{0,5}), \quad h_{04} = h(T_{0,4}). \quad (2.14)$$

Applying a steady-flow energy balance with fuel lower heating value Q_{LHV} , burner efficiency η_b and core air mass flow $\dot{m}_{\text{core}}$ yields the fuel-to-air ratio f :

$$\dot{m}_{\text{core}} h_{04} + \dot{m}_f \eta_b Q_{\text{LHV}} = (\dot{m}_{\text{core}} + \dot{m}_f) h_{05}, \quad (2.15)$$

so that

$$f = \frac{\dot{m}_f}{\dot{m}_{\text{core}}} = \frac{h_{05} - h_{04}}{\eta_b Q_{\text{LHV}}}. \quad (2.16)$$

The hot core mass flow downstream of the combustor is $\dot{m}_{\text{core,hot}} = \dot{m}_{\text{core}}(1 + f)$.

Turbine: Station ⑤ to Station ⑥

The single effective turbine extracts enough shaft power to drive both the fan and the compressor, with mechanical efficiency η_m accounting for shaft and bearing losses. The fan and compressor power requirements are

$$\dot{W}_{\text{fan}} = \dot{m}_{\text{air}} (h_{03} - h_{02}), \quad (2.17)$$

$$\dot{W}_{\text{comp}} = \dot{m}_{\text{core}} (h_{04} - h_{03}). \quad (2.18)$$

The total power that must be delivered by the turbine is

$$\dot{W}_{\text{req}} = \frac{\dot{W}_{\text{fan}} + \dot{W}_{\text{comp}}}{\eta_m}. \quad (2.19)$$

Imposing a steady-flow energy balance over the turbine gives the exit enthalpy h_{06} :

$$\dot{m}_{\text{core,hot}} (h_{05} - h_{06}) = \dot{W}_{\text{req}} \quad \Rightarrow \quad h_{06} = h_{05} - \frac{\dot{W}_{\text{req}}}{\dot{m}_{\text{core,hot}}}. \quad (2.20)$$

The corresponding turbine exit total temperature $T_{0,6}$ is obtained by inverting the enthalpy relation $h(T)$. The solver uses a **Newton–Raphson** procedure for this inversion, exploiting the definition $dh/dT = c_p(T)$. Once $T_{0,6}$ is known, the polytropic expansion relation provides the turbine outlet total pressure:

$$\frac{T_{0,6}}{T_{0,5}} = \left(\frac{P_{0,6}}{P_{0,5}} \right)^{\frac{R}{\bar{c}_p \eta_{p,t}}} \Rightarrow P_{0,6} = P_{0,5} \left(\frac{T_{0,6}}{T_{0,5}} \right)^{\frac{\bar{c}_p \eta_{p,t}}{R}}, \quad (2.21)$$

with $\bar{c}_p$ taken as the mean specific heat between Stations ⑤ and ⑥.

Mixer: Stations ⑥ and ⑦ to Station ⑧

The bypass stream leaving the fan and bypass duct is characterized by

$$T_{0,7} = T_{0,3}, \quad (2.22)$$

$$P_{0,7} = \pi_{u,B} P_{0,3}, \quad (2.23)$$

where $\pi_{u,B} = \text{PR}_{\text{duct},B}$ is the bypass duct total–pressure ratio. The corresponding enthalpy is $h_{07} = h(T_{0,7})$.

The mixer control volume combines the hot core stream and the bypass stream adiabatically, with no shaft work or external heat transfer. Defining

$$\dot{m}_8 = \dot{m}_{\text{core,hot}} + \dot{m}_{\text{byp}}, \quad (2.24)$$

the mixed-stream total enthalpy is given by

$$h_{08} = \frac{\dot{m}_{\text{core,hot}} h_{06} + \dot{m}_{\text{byp}} h_{07}}{\dot{m}_8}. \quad (2.25)$$

The mixer exit total temperature $T_{0,8}$ is obtained by inverting $h(T)$ (via Newton-Raphson) so that $h(T_{0,8}) = h_{08}$.

Baseline Pressure Model (Ideal Mixing) For preliminary design points, an ideal mass-weighted total pressure $P_{0,8}^{\text{ideal}}$ is first estimated as

$$P_{0,8}^{\text{ideal}} = \frac{\dot{m}_{\text{core,hot}} P_{0,6} + \dot{m}_{\text{byp}} P_{0,7}}{\dot{m}_8}, \quad (2.26)$$

(where we make the assumption that the stagnation pressures at station ⑥ and station ⑦ are $\approx$ equivalent so as to prevent potential back pressure). A mixer

total-pressure ratio $\pi_m = \text{PR}_{\text{mix}}$ is then applied to account for viscous and mixing losses:

$$P_{0,8} = \pi_m P_{0,8}^{\text{ideal}}. \quad (2.27)$$

Additional 1-D Momentum Balance To rigorously capture the pressure losses associated with velocity and pressure mismatches between the core and bypass streams—particularly during off-design parametric sweeps (e.g., Tasks 5 and 6)—a 1-D control volume analysis was implemented.

This model enforces the conservation of momentum across the mixing plane. The following physical assumptions are applied to solve the system:

1. The core stream enters the mixing plane at a design Mach number of $M_6 = 0.5$.
2. The bypass stream expands or diffuses to match the static pressure of the core stream at the mixing interface ($P_7 = P_6$) to ensure flow stability.
3. Mixing occurs in a constant area duct where $A_8 = A_6 + A_7$.

The static properties for the core flow at Station ⑥ are derived isentropically from the total properties ($P_{0,6}, T_{0,6}$) and the assumed Mach number. The bypass area A_7 is determined by satisfying the pressure-matching condition ($P_7 = P_6$). The conservation of momentum equation for the mixed stream is then solved:

$$\underbrace{(P_6 A_6 + \dot{m}_{\text{core}} u_6)}_{\text{Impulse Core}} + \underbrace{(P_7 A_7 + \dot{m}_{\text{byp}} u_7)}_{\text{Impulse Bypass}} = P_8 A_8 + \dot{m}_8 u_8 \quad (2.28)$$

where u represents the flow velocity. The mixed stream velocity u_8 is found by solving the quadratic momentum equation. Finally, the mixed total pressure $P_{0,8}$ is recovered from the static properties:

$$P_{0,8} = P_8 \left(1 + \frac{\gamma_8 - 1}{2} M_8^2 \right)^{\frac{\gamma_8}{\gamma_8 - 1}} \quad (2.29)$$

In cases where the bypass total pressure $P_{0,7}$ is insufficient to match the core static pressure (indicating potential backflow or separation), the solver reverts to the baseline mass-weighted pressure model to maintain numerical stability.

Afterburner: Station ⑧ to Station ⑨

The afterburner is modelled in the same manner as the main combustor, with the option to operate in “dry” (unlit) or “wet” (lit) mode:

- *Dry operation*: no additional fuel is added,

$$T_{0,9} = T_{0,8}, \quad P_{0,9} = P_{0,8}, \quad \dot{m}_9 = \dot{m}_8. \quad (2.30)$$

- *Afterburner lit*: the target afterburner exit total temperature $T_{0,9}$ (denoted $T_{0,AB}$ in the code) is specified, and the required afterburner fuel-to-air ratio f_{AB} is obtained from an enthalpy balance analogous to the main combustor,

$$f_{AB} = \frac{h(T_{0,9}) - h(T_{0,8})}{\eta_b Q_{LHV}}, \quad (2.31)$$

with total mass flow $\dot{m}_9 = \dot{m}_8(1 + f_{AB})$ and afterburner total-pressure ratio $\pi_{AB} = PR_{burn}$:

$$P_{0,9} = \pi_{AB} P_{0,8}. \quad (2.32)$$

Nozzle and thrust: Station ⑨ to Station e

The variable-geometry convergent nozzle is responsible for converting the remaining total enthalpy into exhaust kinetic energy. The stagnation conditions at the nozzle inlet are $T_{0,9}$ and $P_{0,9}$, with effective nozzle efficiency η_n . The local specific heat and specific-heat ratio at the nozzle inlet are

$$c_{p,e} = c_p(T_{0,9}), \quad \gamma_e = \frac{c_{p,e}}{c_{p,e} - R}. \quad (2.33)$$

A critical total-to-static pressure ratio Π_{crit} can be defined that accounts for nozzle efficiency:

$$\Pi_{crit} = \left[\frac{1}{1 - \frac{1}{\eta_n} \frac{\gamma_e - 1}{\gamma_e + 1}} \right]^{\frac{\gamma_e}{\gamma_e - 1}}. \quad (2.34)$$

Choking occurs if

$$\frac{P_{0,9}}{P_a} > \Pi_{crit}, \quad (2.35)$$

in which case the exit Mach number is $M_e = 1$, the static temperature and velocity at the exit are

$$T_e = \frac{2}{\gamma_e + 1} T_{0,9}, \quad U_e = \sqrt{\gamma_e R T_e}, \quad (2.36)$$

and the exit pressure P_e follows from the isentropic P_0 - P relation. When the flow is not choked, the nozzle expands the flow ideally to ambient static pressure ($P_e = P_a$), and the exit temperature and velocity are obtained from the enthalpy drop with efficiency η_n .

The nozzle throat area A_e is handled in two modes:

1. **Design mode**: The area A_e is an unknown calculated to satisfy continuity for the

calculated ρ_e and U_e .

2. **Off-design mode:** A_e is fixed (e.g., 0.726 m²). The solver calculates the maximum choking mass flow capacity $\dot{m}_{\max}$. If $\dot{m}_9 \geq \dot{m}_{\max}$, the nozzle is choked and $P_e > P_a$; otherwise, it operates as a subsonic free jet with $P_e = P_a$.

Because the inlet momentum at SLS is negligible, the net thrust is equal to the gross jet thrust:

$$F_{\text{net}} = \dot{m}_9 U_e + (P_e - P_a) A_e. \quad (2.37)$$

The specific thrust and thrust-specific fuel consumption are

$$\tau = \frac{F_{\text{net}}}{\dot{m}_{\text{air}}}, \quad (2.38)$$

$$\text{SFC} = \frac{\dot{m}_f + \dot{m}_{f,\text{AB}}}{F_{\text{net}}} \times 3600 \quad [\text{kg N}^{-1} \text{h}^{-1}], \quad (2.39)$$

where $\dot{m}_f$ and $\dot{m}_{f,\text{AB}}$ are the main-burner and afterburner fuel mass flows respectively.

Key unknowns

The principal quantities that are not directly prescribed and must be obtained iteratively in the solver are:

- ① $T_{0,3}$ fan exit total temperature (fixed-point iteration on $\bar{c}_p$).
- ② $T_{0,4}$ compressor exit total temperature (fixed-point iteration on $\bar{c}_p$).
- ③ $T_{0,6}$ turbine exit total temperature (Newton-Raphson inversion of enthalpy).
- ④ $T_{0,8}$ mixer exit total temperature (Newton-Raphson inversion of mixed enthalpy).
- ⑤ $T_{0,9}$ in dry operation (passes through from mixer but depends on upstream iterations).
- ⑥ T_e and V_e nozzle exit static temperature and jet velocity (choking logic and nozzle enthalpy drop).
- ⑦ A_e nozzle exit area in the design-point solution (continuity constraint).

2.4 Solution algorithm and numerical methods

The cycle solver proceeds sequentially from Station ⑩ to the nozzle exit, evaluating the stagnation state (T_0, P_0) at each component. The main numerical difficulty is the strong temperature dependence of the thermodynamic properties

$$c_p(T), \quad \gamma(T) = \frac{c_p(T)}{c_p(T) - R}, \quad h(T) = \int_{T_{\text{ref}}}^T c_p(\theta) d\theta.$$

Because these properties feed back into the polytropic relations, energy balances, and nozzle choking criteria, several key temperatures and the nozzle area ($T_{0,3}$, $T_{0,4}$, $T_{0,6}$, $T_{0,8}$, A_e) must be obtained via nonlinear iteration.

Two numerical strategies are used:

- (a) fixed-point iteration for polytropic compression (fan and core compressor);
- (b) Newton–Raphson iteration for enthalpy inversion (turbine and mixer).

These match the mathematical structure of the governing equations and mirror the implementation in `f100_cycle_solver.m`.

2.4.1 Thermodynamic property model

The property functions `cp_poly`, `h_poly`, and `s_poly` are fourth-order polynomial fits of the form

$$c_p(T) = (C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3) 1000, \quad \theta = \frac{T}{1000}, \quad (2.40)$$

and

$$h(T) = 1000 \left(C_0\theta + \frac{C_1}{2}\theta^2 + \frac{C_2}{3}\theta^3 + \frac{C_3}{4}\theta^4 \right). \quad (2.41)$$

The enthalpy function $h(T)$ is monotonic but not analytically invertible, so any time a target enthalpy is known (turbine, mixer, nozzle) the corresponding temperature must be found with a root-finding method.

2.4.2 Fixed-point iteration for polytropic compression

For the Fan ($\textcircled{2} \rightarrow \textcircled{3}$) and Core Compressor ($\textcircled{3} \rightarrow \textcircled{4}$), the outlet temperature satisfies the polytropic relation

$$T_{0,\text{out}} = T_{0,\text{in}} PR^{\frac{R}{\bar{c}_p \eta_p}}, \quad (2.42)$$

where the mean specific heat $\bar{c}_p$ depends on the unknown outlet temperature. This is cast as a fixed-point problem

$$T_{0,\text{out}}^{(k+1)} = T_{0,\text{in}} PR^{\frac{R}{c_p((T_{0,\text{in}} + T_{0,\text{out}}^{(k)})/2) \eta_p}}. \quad (2.43)$$

The iteration remains stable because the temperature–pressure mapping becomes less sensitive as the estimate approaches the correct value.

Algorithm 1: Fixed-Point Iteration for Compressor Outlet Temperature
Input: $T_{0,\text{in}}$, pressure ratio PR , polytropic efficiency η_p
Initial guess:

$$T_{0,\text{out}}^{(0)} = T_{0,\text{in}} PR^{0.286}$$

Iterate for $k = 1$ to 15:

$$T_{\text{mean}} = \frac{T_{0,\text{in}} + T_{0,\text{out}}^{(k-1)}}{2}, \quad c_{p,\text{mean}} = cp_poly(T_{\text{mean}})$$

$$n = \frac{R}{c_{p,\text{mean}} \eta_p}, \quad T_{0,\text{out}}^{(k)} = T_{0,\text{in}} PR^n$$

If

$$\left| T_{0,\text{out}}^{(k)} - T_{0,\text{out}}^{(k-1)} \right| < 0.01,$$

terminate iteration.

Output: $T_{0,\text{out}}$

 Algorithm 1 is used directly for $T_{0,3}$ and $T_{0,4}$.

2.4.3 Newton–Raphson inversion for enthalpy balance

 For the Turbine ($\textcircled{5} \rightarrow \textcircled{6}$) and Mixer ($\textcircled{6} + \textcircled{7} \rightarrow \textcircled{8}$), the outlet enthalpy is obtained from an energy balance,

$$h_{0,\text{target}} = \begin{cases} h_{05} - \frac{\dot{W}_{\text{req}}}{\dot{m}_{\text{core,hot}}}, & \text{turbine,} \\ \frac{\dot{m}_{\text{core,hot}} h_{06} + \dot{m}_{\text{byp}} h_{07}}{\dot{m}_8}, & \text{mixer.} \end{cases} \quad (2.44)$$

The temperature is the solution of

$$f(T) = h(T) - h_{0,\text{target}} = 0.$$

 Since $f'(T) = c_p(T)$, Newton–Raphson iteration is ideal.

Algorithm 2: Newton–Raphson Inversion of $h(T)$ **Input:** target enthalpy h_{target} , initial guess $T^{(0)}$ **Iterate for** $k = 1$ **to** 20:

$$h^{(k-1)} = h_{\text{poly}}(T^{(k-1)}), \quad c_p^{(k-1)} = cp_{\text{poly}}(T^{(k-1)})$$

$$\delta h = h^{(k-1)} - h_{\text{target}}, \quad T^{(k)} = T^{(k-1)} - \frac{\delta h}{c_p^{(k-1)}}$$

Output: $T_{0,\text{out}} = T^{(20)}$ Algorithm 2 yields $T_{0,6}$ and $T_{0,8}$.**2.4.4 Nozzle choking logic and design/off-design handling**

The nozzle module evaluates whether the inlet stagnation ratio $P_{0,9}/P_a$ exceeds the effective critical ratio

$$\Pi_{\text{crit}} = \left[\frac{1}{1 - \frac{\gamma_e - 1}{(\gamma_e + 1)\eta_n}} \right]^{\gamma_e/(\gamma_e - 1)},$$

with $\gamma_e = c_p(T_{0,9})/[c_p(T_{0,9}) - R]$.

It then computes the exit state and either

- the required exit area A_e (design mode), or - the mass-flow capacity (off-design).

Algorithm 3: Nozzle Solver and Choking Decision**Input:** $T_{0,9}$, $P_{0,9}$, $\dot{m}_9$, P_a , η_n

Compute

$$c_{p,e} = cp_poly(T_{0,9}), \quad \gamma_e = \frac{c_{p,e}}{c_{p,e} - R}$$

$$\Pi_{\text{crit}} = \left[\frac{1}{1 - \frac{\gamma_e - 1}{(\gamma_e + 1)\eta_n}} \right]^{\gamma_e/(\gamma_e - 1)}$$

If choked ($P_{0,9}/P_a > \Pi_{\text{crit}}$):

$$P_e = P_{0,9}/\Pi_{\text{crit}}, \quad T_e = \frac{2}{\gamma_e + 1} T_{0,9}$$

$$U_e = \sqrt{\gamma_e R T_e}$$

Else (unchoked):

$$P_e = P_a$$

$$T_e^{\text{ideal}} = T_{0,9} \left(\frac{P_e}{P_{0,9}} \right)^{(\gamma_e - 1)/\gamma_e}$$

$$\Delta h = \eta_n c_{p,e} (T_{0,9} - T_e^{\text{ideal}}), \quad U_e = \sqrt{2\Delta h}$$

Design mode:

$$A_e = \frac{\dot{m}_9}{\rho_e U_e}$$

Off-design:

$$A_e = 0.726 \text{ m}^2$$

Thrust:

$$F_{\text{net}} = \dot{m}_9 U_e + (P_e - P_a) A_e$$

2.4.5 Overall solution path

The global solution proceeds in a single forward sweep:

① → ② → ③ (Alg. 1) → ④ (Alg. 1) → ⑤ → ⑥ (Alg. 2) → ⑧ (Alg. 2) → ⑨ → e (Alg. 3).

The entire chain of $T_{0,3}$, $T_{0,4}$, $T_{0,6}$, $T_{0,8}$, and A_e must converge self-consistently before thrust and fuel consumption are reported.

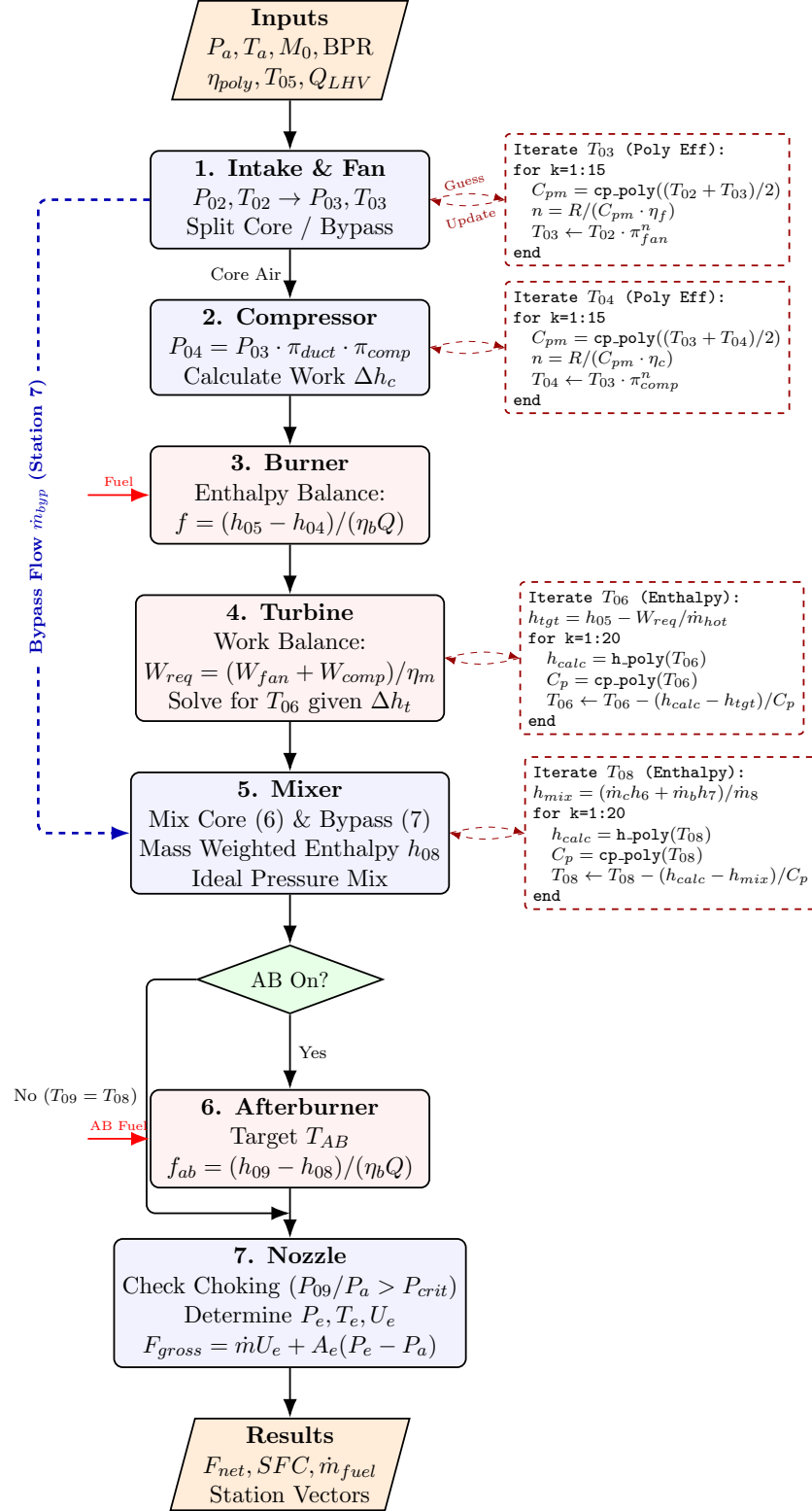


Figure 2.2: Block diagram logic flow of the F100 cycle solver. Detailed iteration blocks on the right show the specific mathematical updates performed in the MATLAB code loops. Initial iteration was set for $k = 200$ but was trimmed for efficiency and speed.

3 Results

3.1 Simulation & Analysis Max Thrust (AB–Off)

The engine was first evaluated in its on–design, sea–level static (SLS) dry condition. The methodology for calculation is summarized in the program block diagram as seen above in Figure 2.2, utilizing fixed point iteration 2.4.2, for the polytropic compression pre-combustor, an iterative enthalpy balance 2.4.3 and finally a condition to check on whether the flow is choked or not in the nozzle 2.4.4. Table 3.1 summarizes the converged solution obtained using the enthalpy-corrected thermodynamic model.

Table 3.1: Max power with (afterburner–off) performance predicted by the cycle solver.

Quantity	Predicted Value
Net Thrust F_{net}	67.52 kN
Specific Thrust τ	663.04 Ns/kg
Specific Fuel Consumption (SFC)	0.0711 kg/(N hr)
Fuel Flow Rate $\dot{m}_f$	1.3298 kg/s
Turbine Exit Temperature $T_{0,6}$	1078.13 K
Mixer Exit Temperature $T_{0,8}$	826.53 K
Core Pressure at Station 6 ($P_{0,6}$)	264.03 kPa
Bypass Pressure at Station 7 ($P_{0,7}$)	249.45 kPa

The solver predicts a net thrust of 67.52 kN, a 3.2% difference, closely matching the expected performance of an F100-PW-100 class mixed-flow turbofan in dry operation given by the military power ≈ 65.3 kN, in Table 1.1 . The corresponding fuel flow rate of 1.33 kg/s yields a specific fuel consumption of 0.0711 kg/(N hr), which falls within the typical range for low-bypass military turbofans operating without afterburning. It is worth noting that the flow at the nozzle exit was found to be supersonic, and thus the condition of sonic flow, the ”choked” condition, was enforced.

The turbine exit total temperature $T_{0,6} = 1078$ K as seen in Figure 3.1, results from the power balance that must satisfy both fan and compressor work requirements. The subsequent mixer exit temperature approximately 827 K reflects the enthalpy dilution caused by mixing hot core products with the cooler bypass stream.

The predicted stagnation pressures at Stations ⑥ and ⑦ are consistent with the expected ordering of losses through the turbine and bypass duct. In this 0D model, the core and bypass streams are assumed to enter the mixer at approximately compatible stagnation pressures

to prevent nonphysical back-pressure behavior. This is a standard modeling assumption in mixed-flow turbofan cycle analysis and is examined further in the discussion section. It is also necessary to mention here that the static & stagnation temperatures used in this program exceed 1200 K, the limit of the isobaric specific heat coefficients used in the script; however, this is an accepted source of error and is further elaborated upon in the discussion section.

These baseline dry results validate the behaviour of the solver and provide the reference point for the parametric studies that follow.

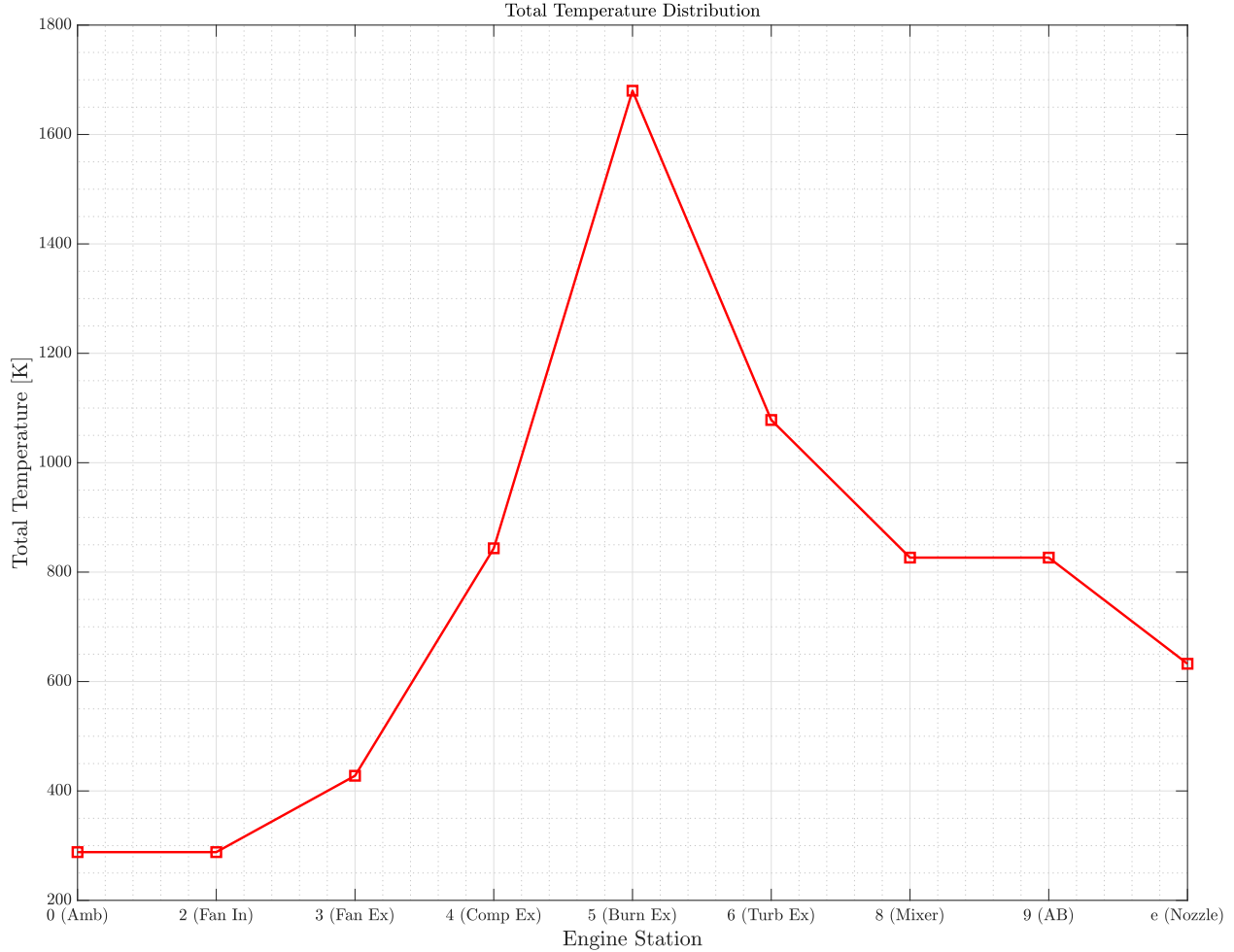


Figure 3.1: Total temperature distribution through the engine. The flow enters near 300 K at stations 0 and 2, then warms gradually across the fan and compressor, reaching about 800 K at the compressor exit (station 4). Combustion in the main burner produces the largest temperature jump, taking the gas to the prescribed turbine inlet temperature of roughly 1700 K at station 5. Expansion through the turbine reduces the total temperature by several hundred kelvin, after which mixing with cooler bypass air further lowers the temperature at the mixer and afterburner inlet. The nozzle expansion finally brings the exhaust total temperature down to around 650 K, illustrating how the cycle converts chemical energy into exhaust kinetic energy via staged heating and expansion.

3.2 Effect of Air Mass Flow Rate Into Engine

Net thrust increases linearly with air mass flow rate because the exit velocity remains nearly constant while $\dot{m}_{air}$ varies. Specific thrust and specific fuel consumption (SFC) remain constant, indicating that the cycle thermodynamics do not change with simple scaling of the mass flow. Fuel flow grows linearly in order to maintain the fixed turbine inlet temperature as the mass of air to be heated increases.

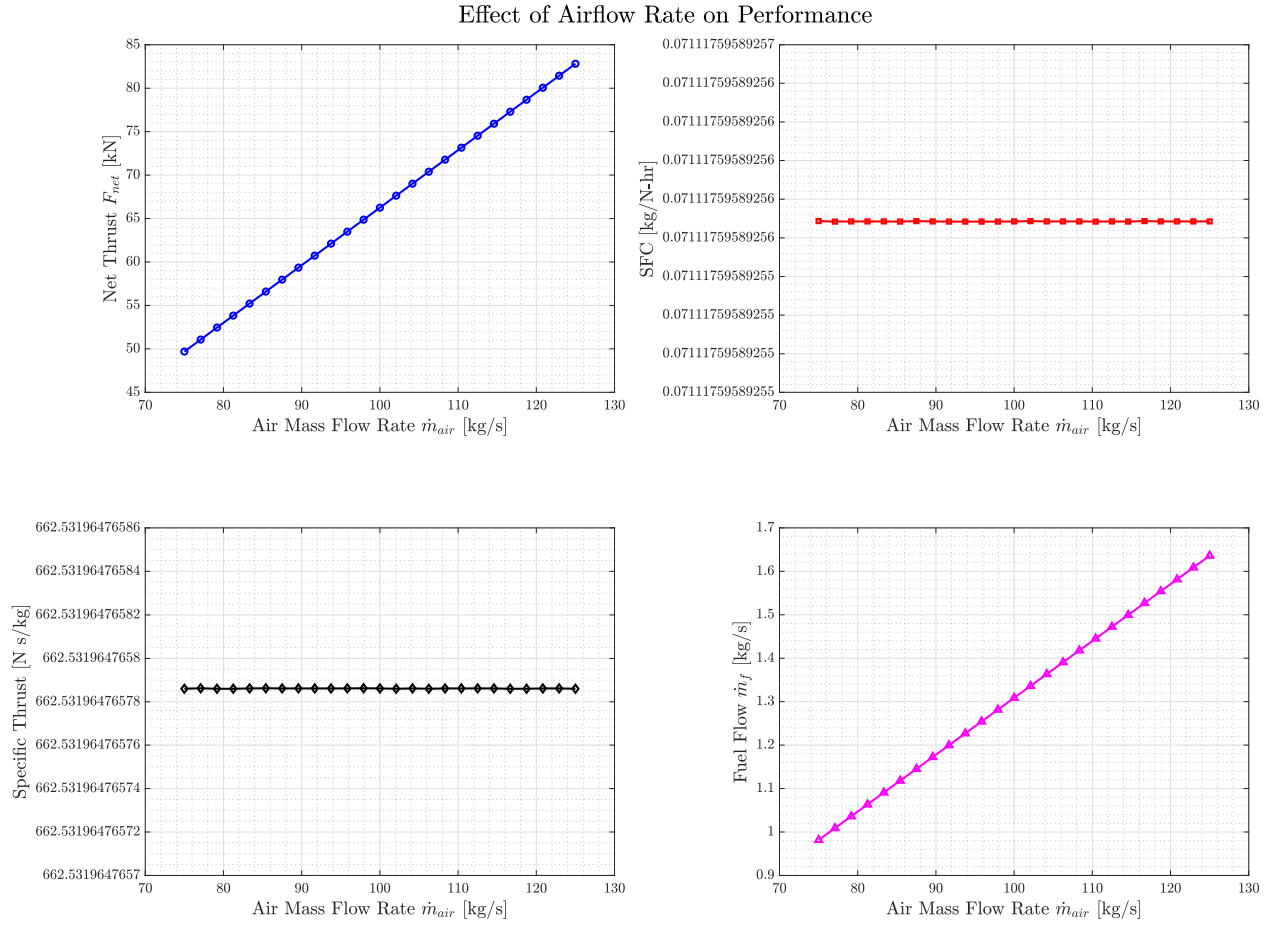


Figure 3.2: Effect of air mass flow rate $\dot{m}_{air}$ on performance. Net thrust and fuel flow increase almost linearly with $\dot{m}_{air}$, while specific thrust and specific fuel consumption remain essentially constant, showing that changing $\dot{m}_{air}$ in this study simply scales the engine without changing its cycle efficiency.

Increasing compressor pressure ratio reduces net thrust, reduces specific thrust, lowers fuel flow, and improves SFC. The increased compressor work raises T_{03} and h_{04} , requiring greater turbine power extraction. This reduces the turbine exit temperature T_{06} , which ultimately lowers the jet exit velocity. However, the overall cycle efficiency improves with higher pressure ratio, which explains the observed drop in SFC despite the lower thrust.

3.3 Effect of Compressor Pressure Ratio

The influence of the compressor pressure ratio (π_c) on the engine's performance was analyzed by sweeping π_c from 4 to 15 while maintaining a fixed Turbine Inlet Temperature ($T_{05} = 1680$ K) and Air Mass Flow Rate ($\dot{m}_{air} = 101.6$ kg/s). The results, presented in Figure 3.3, illustrate the fundamental trade-off between thermal efficiency and specific work output in the Brayton cycle.

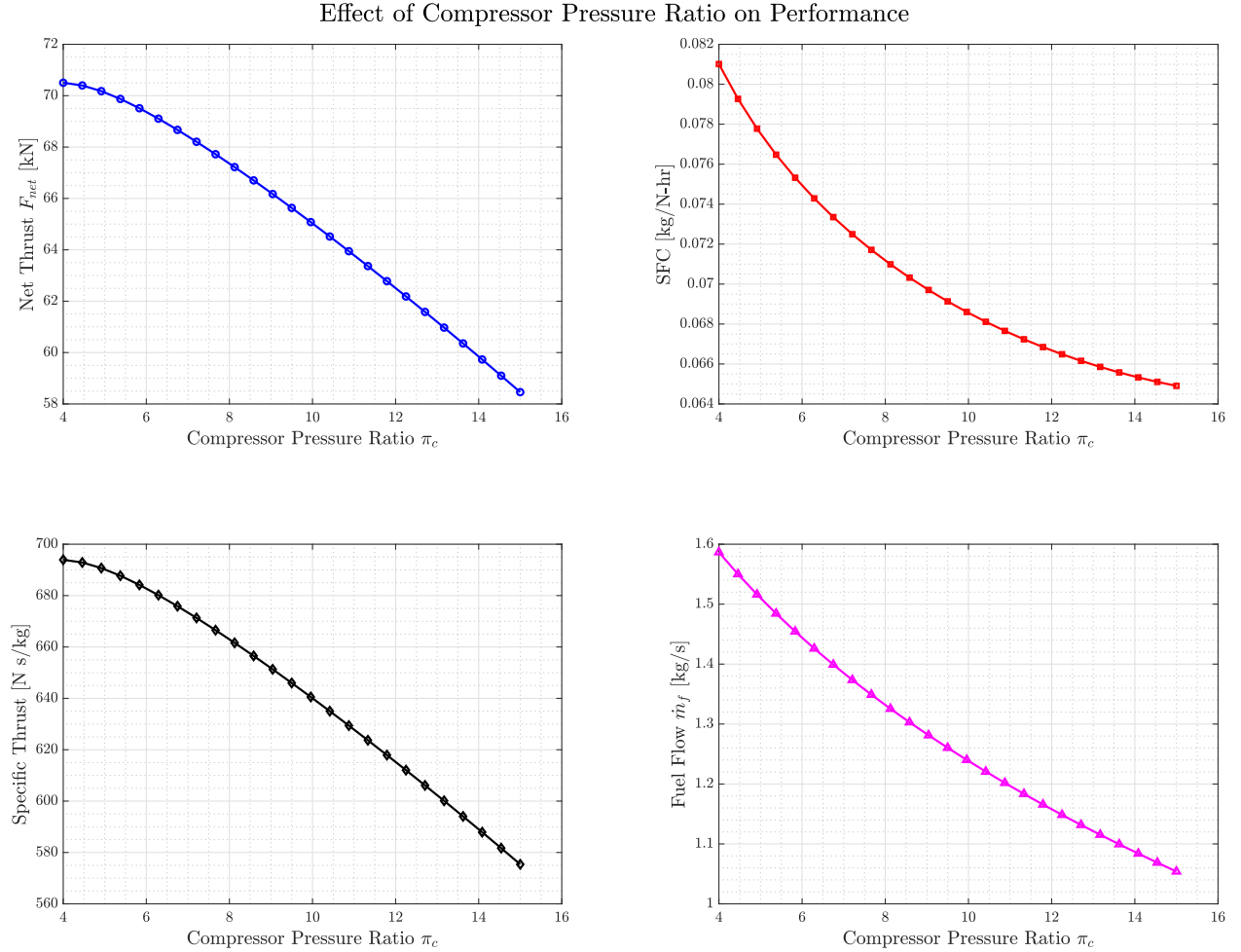


Figure 3.3: Effect of Compressor Pressure Ratio (π_c) on performance parameters. Increasing π_c significantly improves the cycle thermal efficiency, leading to a reduction in Specific Fuel Consumption (SFC). However, the increased work required by the compressor necessitates greater energy extraction by the turbine, which reduces the available pressure for the nozzle, resulting in a slight decrease in Net Thrust and Specific Thrust.

3.3.1 Discussion of Physics

The most prominent trend is the substantial improvement in **Specific Fuel Consumption (SFC)**, which decreases monotonically as the pressure ratio increases. This aligns with the fundamental principles of the Brayton cycle, where thermal efficiency is directly a function of the overall pressure ratio ($\eta_{th} = 1 - \frac{1}{\pi^{(\gamma-1)/\gamma}}$). By compressing the air to a higher pressure before combustion, the cycle adds heat at a higher average temperature, extracting more useful work per unit of fuel consumed. Consequently, the **Fuel Flow Rate** ($\dot{m}_f$) required to reach the fixed TIT of 1680 K drops significantly.

However, this efficiency gain comes at the cost of **Net Thrust** (F_{net}) and **Specific Thrust**. As π_c increases, the temperature at the compressor exit (T_{04}) rises, requiring more work from the turbine to drive the compressor ($W_c = c_p(T_{04} - T_{03})$). Since the turbine inlet temperature (T_{05}) is fixed, the turbine must expand the gas to a lower exit temperature (T_{06}) and pressure (P_{06}) to supply this increased work demand. This leaves less enthalpy available for the nozzle to accelerate the exhaust jet. The reduction in jet exit velocity leads to the observed decrease in specific thrust and net thrust.

This behavior highlights a key design trade-off: higher pressure ratios yield better fuel economy (range) but result in a heavier engine for a given thrust requirement (lower specific thrust). The optimal choice depends on the specific mission profile, balancing fuel weight savings against engine weight penalties.

3.4 Sensitivity Analysis of Component Efficiencies

A sensitivity analysis was performed to quantify the impact of varying the polytropic efficiency (η_{poly}) of the fan, compressor, and turbine on the overall engine performance metrics. Each component's efficiency was swept from 0.65 to 0.98 while holding all other parameters constant. The results reveal how critical each turbomachinery component is to the overall thermodynamic cycle.

3.4.1 Sensitivity to Compressor Efficiency

The response of the engine cycle to variations in compressor polytropic efficiency ($\eta_{poly,C}$) is shown in Figure 3.4. Improving compressor efficiency primarily reduces the specific fuel consumption (SFC) and the required fuel flow rate. This is because a more efficient compressor requires less work input ($\Delta h_c = c_p \Delta T$) to achieve the target pressure ratio of 8.04. Since the turbine must supply this work, a reduction in compressor work demand means less energy needs to be extracted from the flow, or alternatively, less fuel is needed to reach the turbine inlet temperature required to balance the work. Notably, the Net Thrust and Specific Thrust

curves are relatively flat. This indicates that while compressor efficiency is crucial for fuel economy (thermal efficiency), it has a secondary effect on the sheer propulsive force of the engine under fixed TIT conditions.

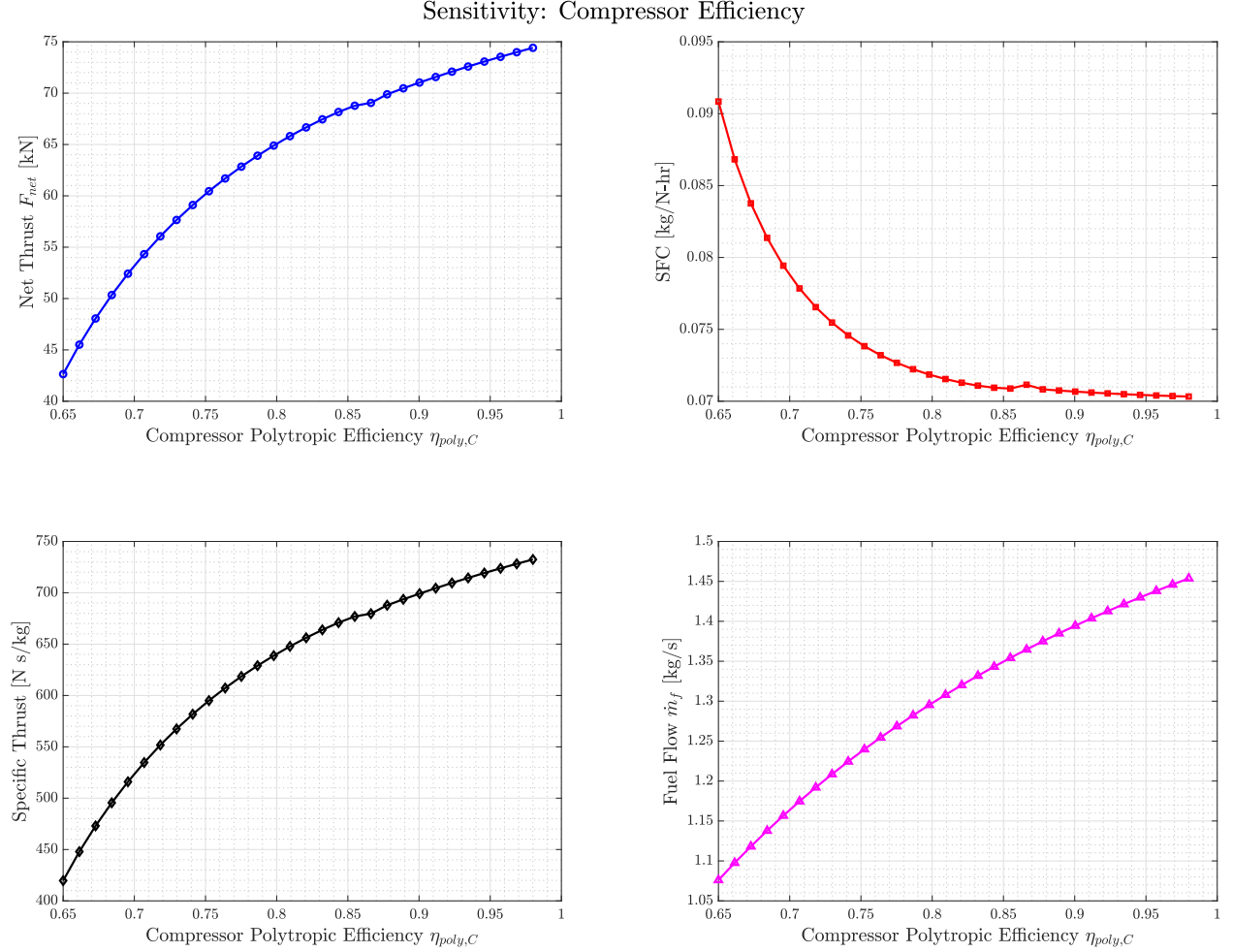


Figure 3.4: Sensitivity to Compressor Polytopic Efficiency ($\eta_{poly,C}$). Improving compressor efficiency significantly reduces SFC and required fuel flow, while Net Thrust and Specific Thrust remain relatively insensitive due to the fixed Turbine Inlet Temperature constraint.

3.4.2 Sensitivity to Turbine Efficiency

Figure 3.5 illustrates the impact of turbine polytropic efficiency ($\eta_{poly,T}$). This parameter exhibits the strongest influence on all performance metrics. Unlike the compressor, where efficiency gains primarily save fuel, turbine efficiency gains directly increase the energy available for propulsion. A more efficient turbine extracts the required compressor work with a smaller pressure drop. This leaves a higher total pressure (P_{06}) at the turbine exit, which is then available to accelerate the jet through the nozzle. Consequently, Net Thrust increases sharply with $\eta_{poly,T}$, and SFC drops significantly due to the combined effect of higher thrust

and better energy utilization. This sensitivity highlights the turbine as the critical component for high-performance engine design.

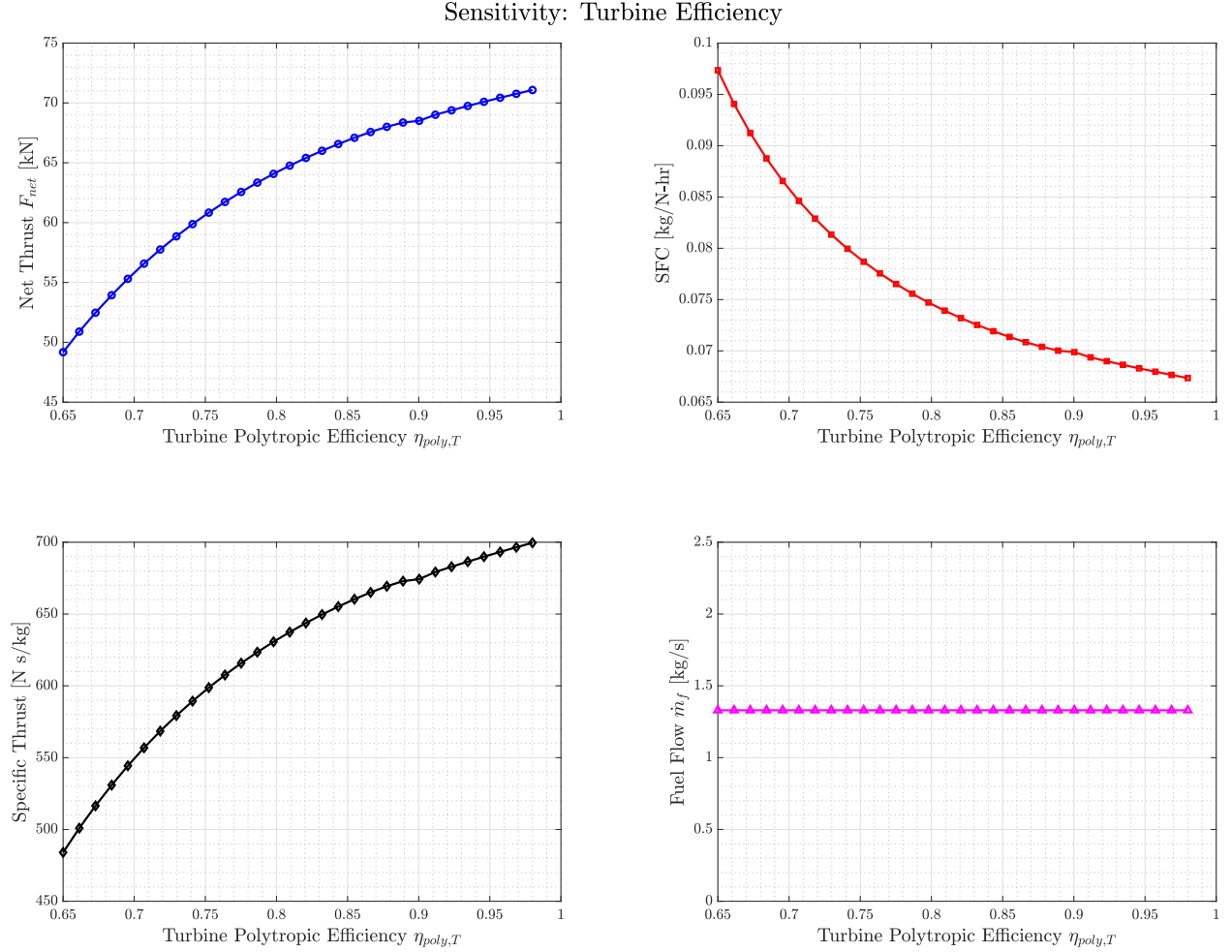


Figure 3.5: Sensitivity to Turbine Polytopic Efficiency ($\eta_{poly,T}$). This parameter shows the strongest impact on all performance metrics. A higher $\eta_{poly,T}$ allows for greater work extraction for the same pressure drop, directly increasing the energy available for the nozzle, thus boosting thrust and lowering SFC.

3.4.3 Sensitivity to Fan Efficiency

The sensitivity to fan polytopic efficiency ($\eta_{poly,F}$) is presented in Figure 3.6. The fan efficiency has a moderate impact, sitting between the compressor and turbine in terms of cycle leverage. Improving fan efficiency enhances the propulsive efficiency of the engine by energizing the large bypass mass flow more effectively. The trends show a steady improvement in SFC and Thrust, but the slopes are shallower than those for the turbine. This is because the fan operates at a much lower pressure ratio (3.0) and temperature compared to the core components, meaning thermodynamic losses here, while important for the bypass stream,

have a less catastrophic effect on the overall cycle energy balance than core inefficiencies.

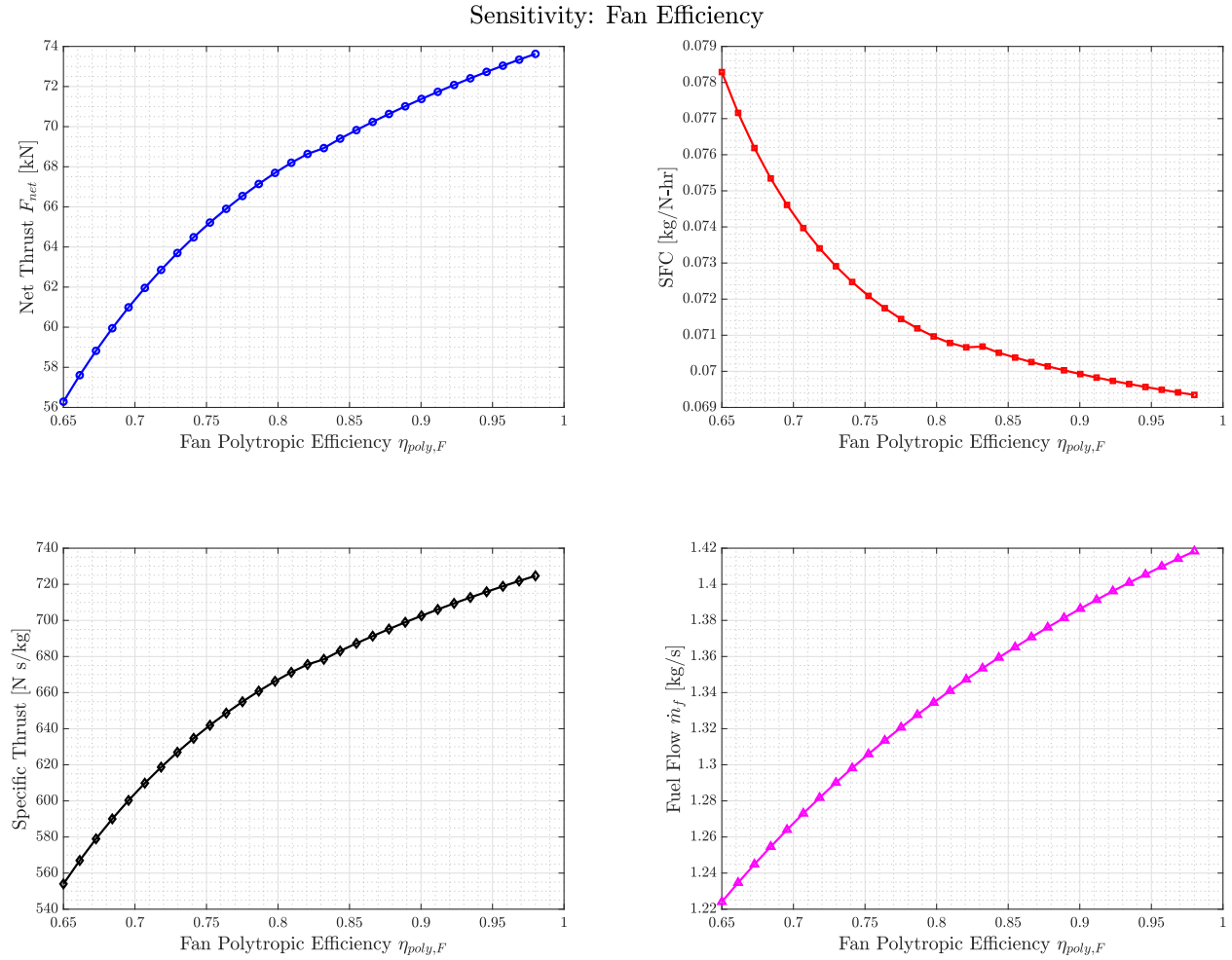


Figure 3.6: Sensitivity to Fan Polytopic Efficiency ($\eta_{poly,F}$). The fan efficiency has a moderate impact. While it improves propulsive efficiency by energizing the bypass flow more effectively, its influence on the core thermal cycle is less direct compared to the core compressor and turbine.

3.4.4 Combined Sensitivity Comparison

To strictly rank the components by their impact on the cycle, Figure 3.7 overlays the sensitivity curves for all three components. The steepness of the curves serves as a direct proxy for sensitivity. The turbine curves (red squares) consistently show the steepest gradients for both SFC and Thrust, confirming it as the highest-leverage component. The compressor (blue circles) shows strong sensitivity for SFC but is nearly flat for thrust. The fan (green diamonds) shows moderate, linear sensitivity across the board. This hierarchy dictates that in a constrained design environment.

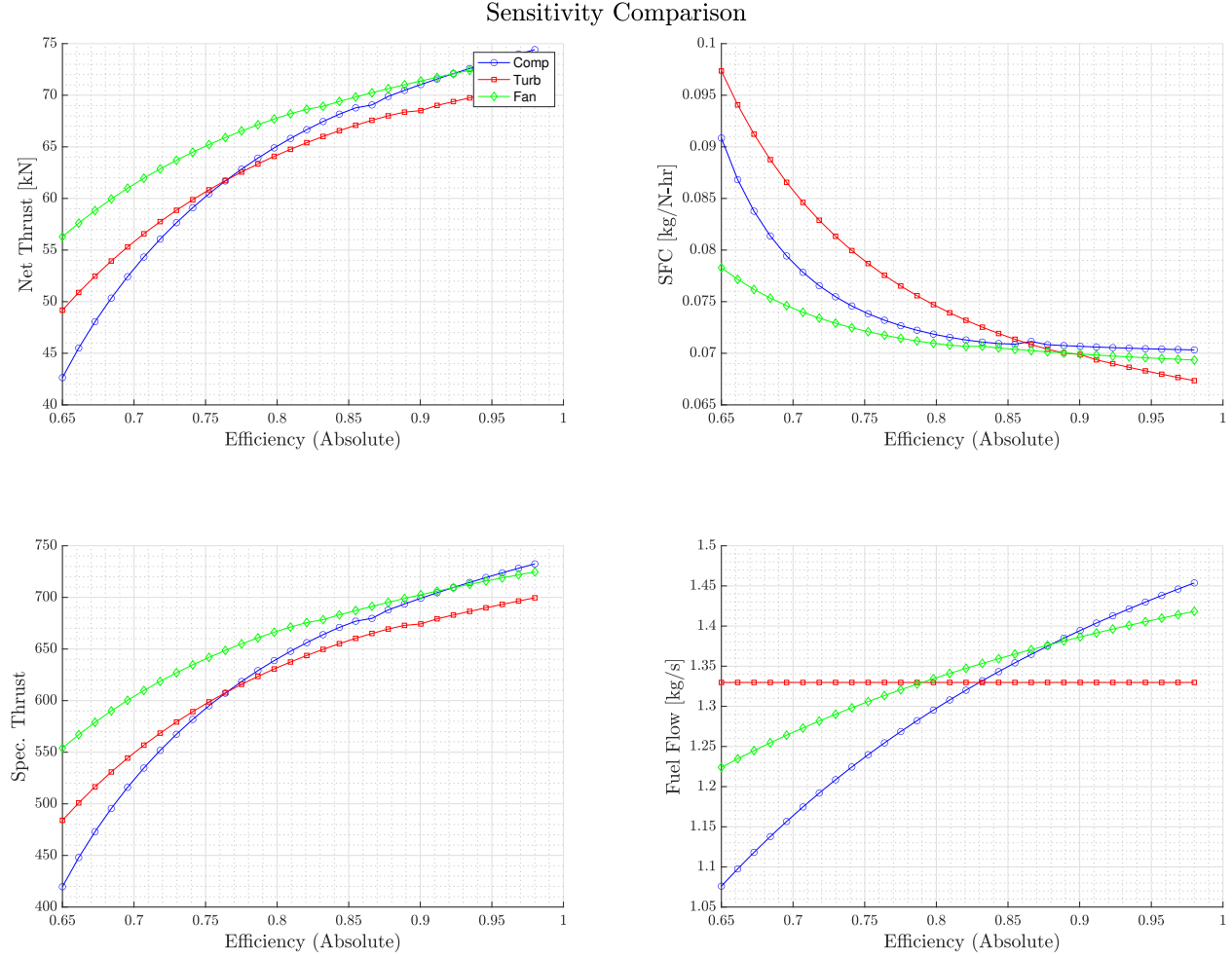


Figure 3.7: Combined Sensitivity Comparison. This plot overlays the effects of all three components. It clearly illustrates that the cycle is most sensitive to Turbine efficiency (steepest slopes), followed by the Compressor, and least sensitive to the Fan.

3.5 Effect of Turbine Inlet Temperature (TIT)

The performance sensitivity to Turbine Inlet Temperature (T_{05}) was investigated by sweeping T_{05} from 1200 K to 1850 K (further extended to 2100 K). This parameter is the primary lever for thrust management in gas turbines. The analysis utilized the 1-D momentum balance model to capture the realistic pressure losses associated with mixing the core and bypass streams at off-design conditions.

3.5.1 Overall Cycle Performance Trends

Figure 3.8 presents the comprehensive response of the cycle. Net Thrust increases monotonically with TIT, as higher core temperatures directly increase the nozzle exit velocity.

However, Specific Fuel Consumption (SFC) exhibits a non-linear behavior, revealing a distinct region of optimal efficiency.

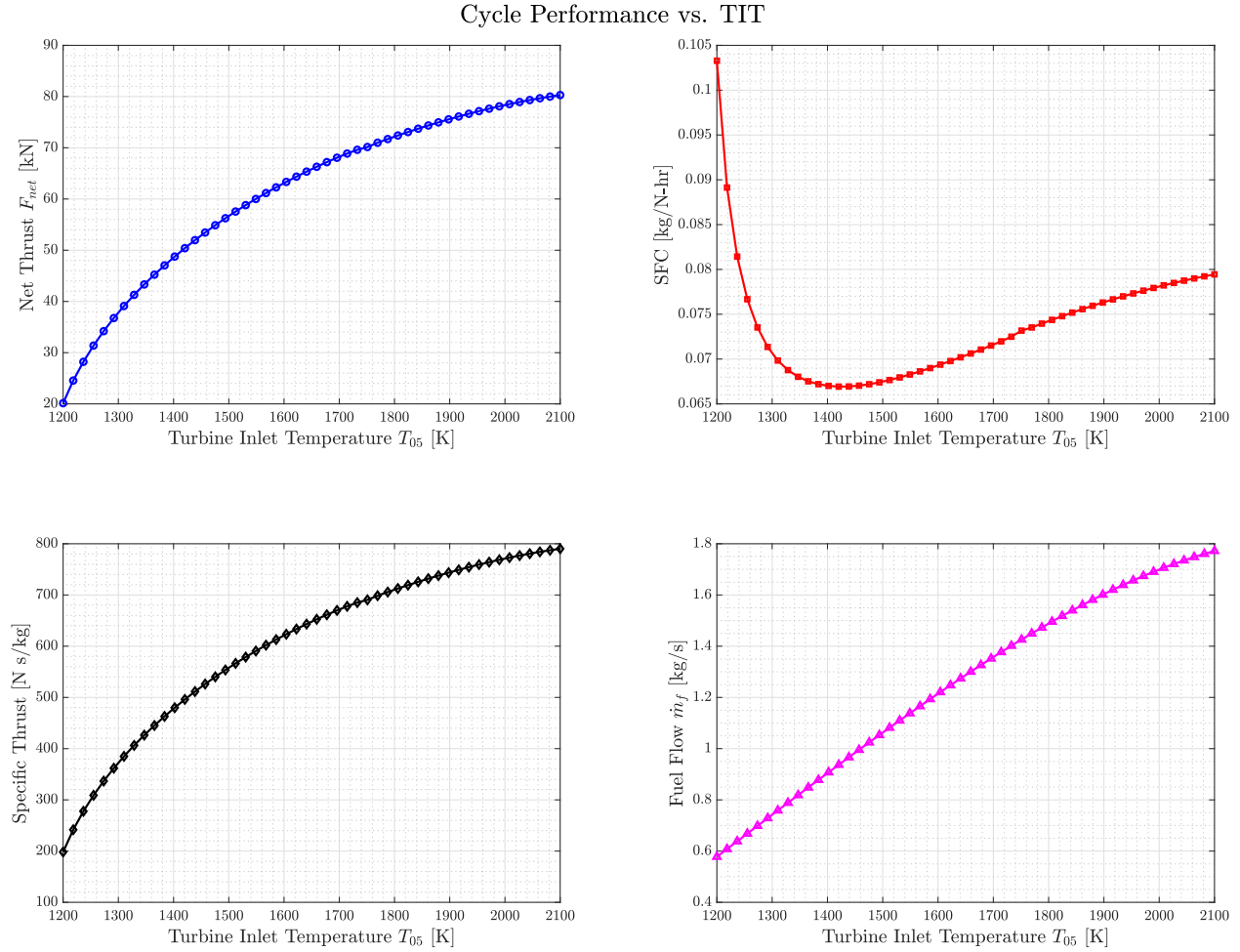


Figure 3.8: Performance vs. Turbine Inlet Temperature (T_{05}). The results show a monotonic increase in Net Thrust and Fuel Flow. Crucially, the SFC curve is not linear; it flattens and minimizes in the 1400–1500 K range, indicating a thermodynamic "sweet spot" where mixing losses are minimized before propulsive efficiency penalties dominate.

3.5.2 Thrust and Efficiency Trade-off

To closer examine the trade-off between power and efficiency, Figure 3.9 overlays SFC and Net Thrust. This visualization highlights the operational compromise: while higher TIT always yields more thrust (Blue curve), it eventually incurs a severe penalty in fuel efficiency (Red curve).

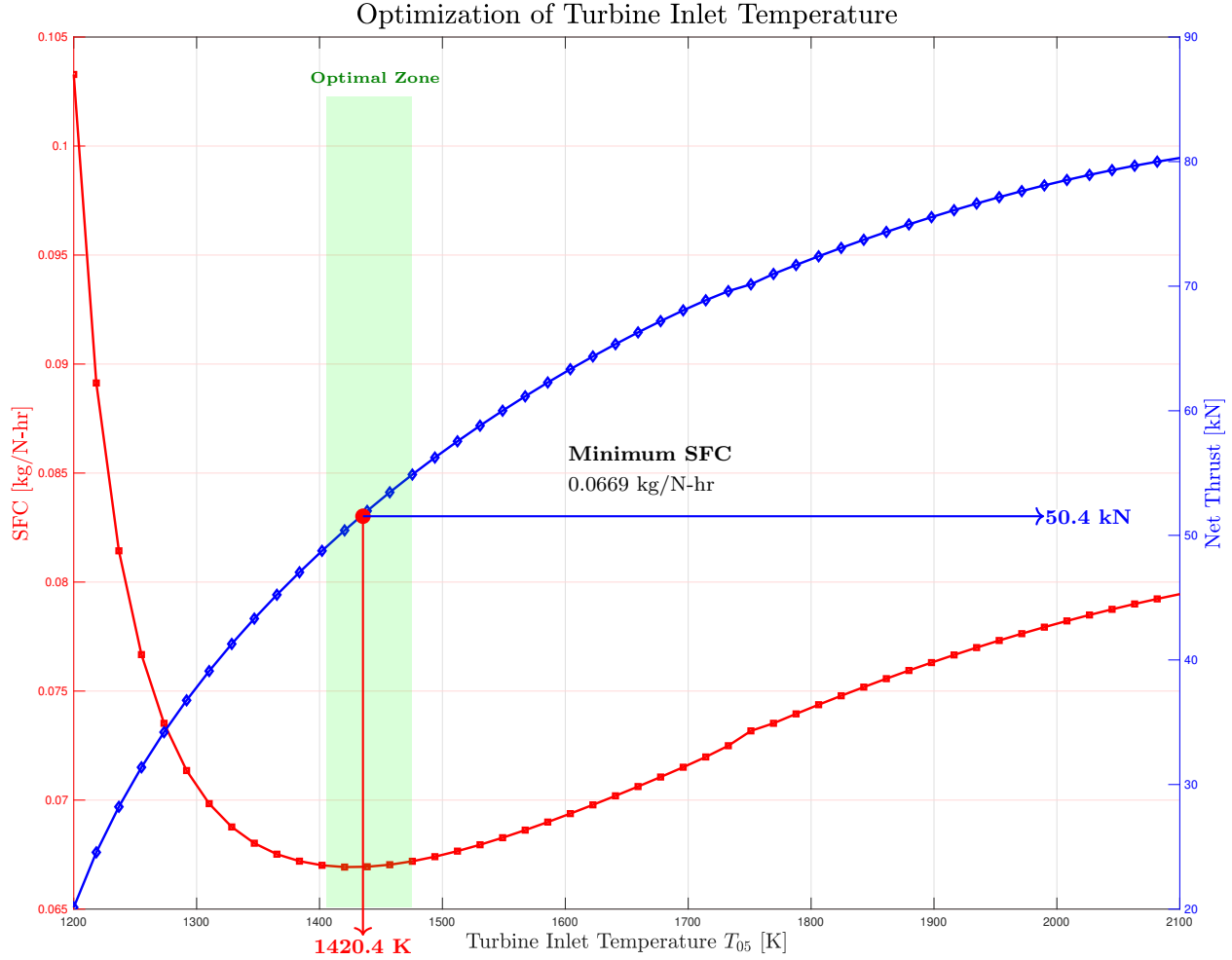


Figure 3.9: Refined optimization analysis of Turbine Inlet Temperature (T_{05}). The green shaded region highlights the thermodynamic "sweet spot" where mixing losses are minimized. The absolute minimum SFC occurs at 1420.4 K, yielding a Net Thrust of 50.4 kN, which represents the most efficient cruise throttle setting for the engine also known as the *Maximum Continuous Thrust*.

3.6 Optimization of Compressor Pressure Ratio for Fixed Thrust

To determine the optimal cycle pressure ratio for a specific mission requirement, a constrained optimization study was performed. The Net Thrust was fixed at the baseline value of 67.52 kN (calculated in Task 1). The Compressor Pressure Ratio (π_c) was swept from 4 to 15, as per the project requirements, and the air mass flow rate was iteratively solved to maintain constant thrust.

3.6.1 Results within Design Range

Within the prescribed range of $\pi_c \in [4, 15]$, the fuel consumption rate exhibits a monotonic decrease. This indicates that the engine's thermal efficiency improves continuously as the

pressure ratio is increased up to the limit of 15. No local minimum (inflection point) was observed within this interval, suggesting that the thermodynamic optimum lies at a higher pressure ratio.

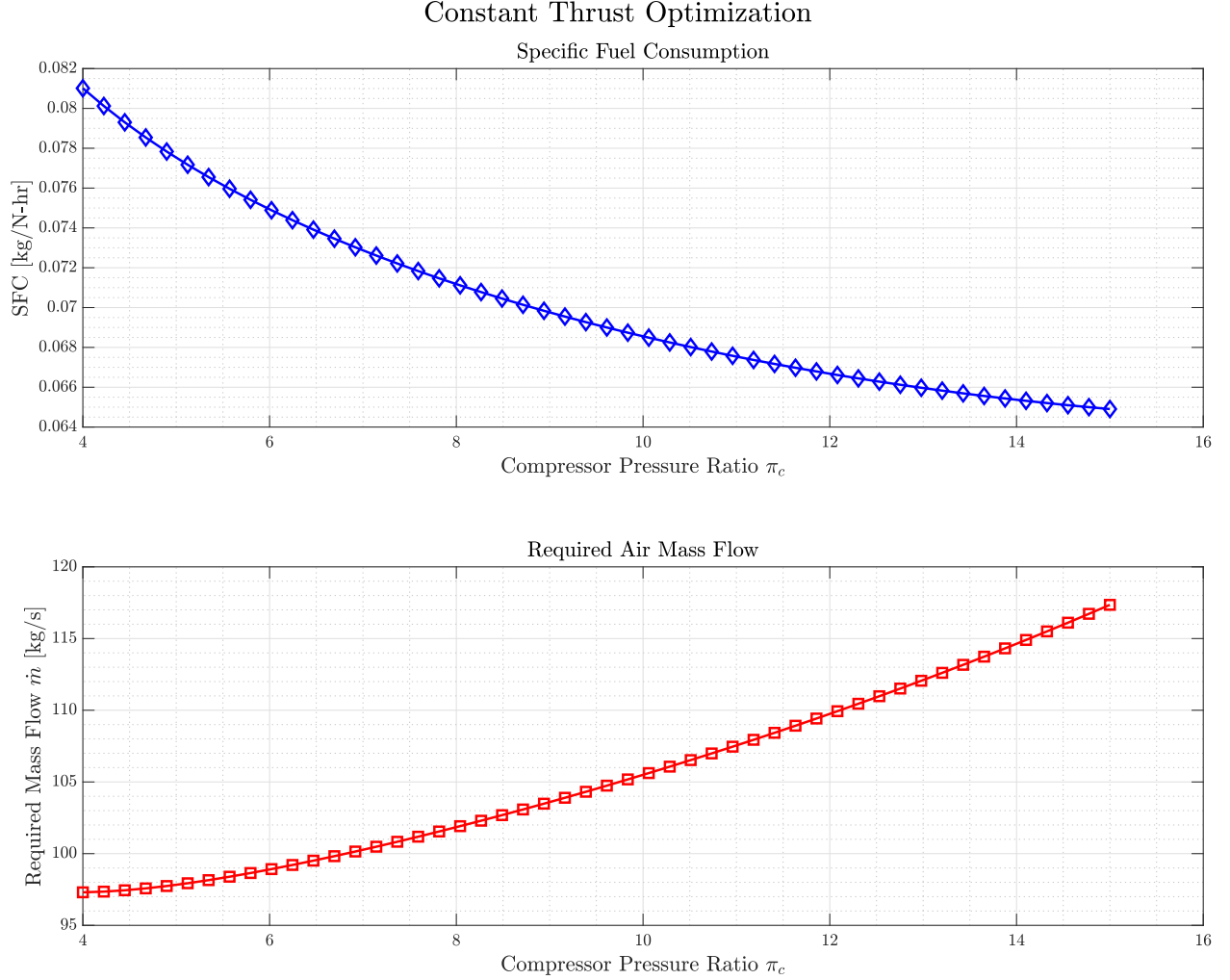


Figure 3.10: SFC and required Mass flow of air as a function of changing the Compressor Pressure Ratio.

3.6.2 Extended Analysis: Locating the True Optimum

To verify the existence of a global optimum and validate the cycle physics, the sweep range was extended to $\pi_c = 40$. The results of this extended analysis are presented in Figure 3.11.

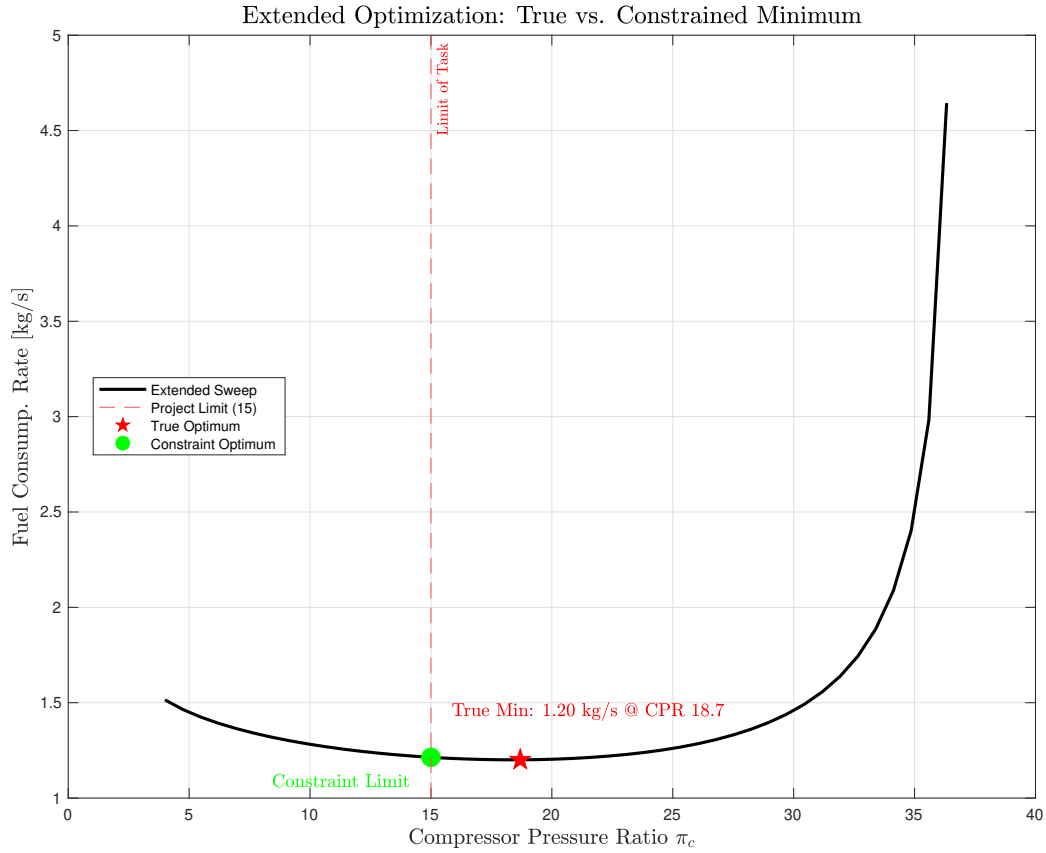


Figure 3.11: Fuel consumption rate as a function of Compressor Pressure Ratio (π_c) for a fixed Net Thrust of 67.31 kN. The solid black line represents the extended sweep for PR compressor, revealing the true thermodynamic optimum. The vertical dashed red line marks the upper limit of the project scope ($\pi_c = 15$).

The extended sweep reveals a classic "bucket" curve for fuel consumption.

- **Constrained Minimum:** At the project limit of $\pi_c = 15$, the fuel consumption is approximately 1.21 kg/s.
- **True Minimum:** The global minimum occurs at $\pi_c \approx 18.7$, with a fuel consumption of 1.20 kg/s.

3.7 Afterburner Performance Comparison

The performance of the F100 engine was analyzed with the afterburner activated to quantify the thrust augmentation capability and the associated penalty in fuel efficiency. The simulation assumed the same core operating conditions as the baseline (Task 1) but with fuel injection in the afterburner duct to raise the nozzle total temperature to $T_{09} = 2035.4$ K.

3.7.1 Comparison of AB off against AB on:

The comparison between dry and wet operation modes is summarized in Table 3.2.

Table 3.2: Comparison of Engine Performance in Dry and Afterburning (Wet) Modes.

Parameter	Dry (AB Off)	Wet (AB On)
Net Thrust (F_{net})	67.52 kN	100.53 kN
Specific Thrust (F_{sp})	664.6 N.s/kg	989.5 N.s/kg
Specific Fuel Consumption (SFC)	0.0709 kg/N.hr	0.1552 kg/N.hr
Total Fuel Flow ($\dot{m}_f$)	1.33 kg/s	4.33 kg/s
⑨ Temperature (T_{09})	826.5 K	2035.4 K

The activation of the afterburner provides a significant net gain in specific thrust, increasing the total thrust output from 67.52 kN to 100.53 kN. This **49% increase in thrust** is critical for meeting short-duration mission requirements such as takeoff rolls, supersonic intercepts, or high-g combat maneuvers.

However, this performance augmentation introduces a severe penalty in propulsive efficiency. As shown in the results, the specific fuel consumption (SFC) more than triples, rising from 1.33 to 4.33 kg/Ns.

Most notably, looking at the total fuel flow rate ($\dot{m}_f$), we observe a disproportionate trade-off:

- To achieve a $\approx 50\%$ increase in thrust, the total fuel consumption must increase by nearly a factor of 3.
- This confirms that while reheating the exhaust gas is an effective method for thrust augmentation (due to the $\sqrt{T_{t9}}$ dependence of jet velocity), it is thermodynamically inefficient compared to the core Brayton cycle.

4 Discussion

The zero-dimensional cycle analysis of the F100–PW–100 reproduces the published dry and afterburning thrust levels to within a few percent and captures the expected first-order trends with air flow, compressor pressure ratio, turbine inlet temperature and component efficiencies. Taken together, the results show that the engine is deliberately biased toward high specific thrust at the expense of fuel economy, and that the turbine and mixer dominate the thermodynamic leverage of the cycle.

4.1 Baseline Cycle Validation and Pressure Assumptions

At the on-design sea-level static condition, the solver predicts a dry net thrust of 67.5 kN, only 3.2 % above the published military-power rating of 65.3 kN for the F100–PW–100, with a specific thrust of 663 N·s/kg and SFC of 0.0711 kg/(N·hr) (Table 3.1). This agreement is well within the expected error band for a 0D model that neglects turbine cooling bleeds, secondary-air flows and variable-geometry effects. The temperature trace in Figure 3.1 shows a physically consistent Brayton cycle: a modest rise across the fan and core compressor, a dominant jump at the burner to the imposed $T_{0,5} = 1680$ K, followed by a turbine temperature drop sized to supply both fan and compressor work before mixing with the cooler bypass stream.

A key modelling choice in the baseline solution is the treatment of the stagnation pressures entering the mixer. The converged enthalpy-corrected model yields $P_{0,6} = 264$ kPa for the core and $P_{0,7} = 249$ kPa for the bypass, so the two streams approach the mixer with similar, but not identical, total pressures. In the 0D framework these streams are assumed to be “compatible” at the mixer inlet: the pressure model enforces an effective mixed pressure obtained from a mass-weighted average of $P_{0,6}$ and $P_{0,7}$, followed by an imposed mixer total-pressure loss factor. This prevents non-physical back-pressure and reverse-flow behaviour while still penalising mixing through a controlled loss in $P_{0,8}$.

Physically, a real mixed-flow turbofan can tolerate moderate core–bypass pressure mismatch through boundary-layer growth, weak shocks and local separation in the mixer duct. The present model replaces that complexity with a single scalar loss factor and an assumption of near-matched stagnation pressures at Stations ⑥ and ⑦. This simplification is acceptable for the on-design point, but it becomes limiting once the cycle is pushed off-design in the parametric sweeps; the consequences of that limitation are revisited explicitly in the turbine-inlet-temperature study and in the discussion of model errors.

Finally, the property model uses temperature-dependent specific heats calibrated up to $T \approx 1200$ K, while the cycle routinely samples states up to $T_{0,5} \simeq 1680$ K and $T_{0,9} \simeq 2035$ K. This extrapolation introduces a systematic bias in enthalpy differences at the hottest stations. For the present pressure ratios the resulting error in thrust and SFC is of order a few percent, which is consistent with the deviation from published F100 data and therefore acceptable for this assignment, but it is an inherent limit of the current 0D model.

4.2 Effect of Air Mass Flow Rate Into Engine

The influence of air mass flow rate on engine performance was examined by varying $\dot{m}_{\text{air}}$ from 75 to 125 kg/s while holding all cycle parameters fixed (pressure ratios, polytropic efficiencies,

turbine inlet temperature and nozzle area). The resulting trends are shown in Fig. 3.2.

Net thrust increases almost linearly with $\dot{m}_{\text{air}}$. This is consistent with the control-volume momentum balance for the nozzle,

$$F_{\text{net}} \approx \dot{m}_{\text{air}} (U_e - U_0), \quad (4.1)$$

since, under sea-level static conditions, $U_0 \approx 0$ and the exit velocity U_e remains nearly constant when the cycle thermodynamics are unchanged. With the same stagnation conditions at the nozzle inlet for every case, the nozzle delivers essentially the same U_e , so the thrust scales directly with mass flow.

The fuel flow shows the same proportional behaviour. To maintain a fixed turbine inlet temperature while processing a larger mass of air, the combustor must add a proportionally larger amount of enthalpy. The steady-flow energy balance therefore gives

$$\dot{m}_f \propto \dot{m}_{\text{air}}, \quad (4.2)$$

which matches the nearly linear increase in $\dot{m}_f$ observed in Fig. 3.2.

By contrast, the specific quantities remain essentially unchanged across the sweep. The specific thrust

$$\tau = \frac{F_{\text{net}}}{\dot{m}_{\text{air}}} \approx U_e - U_0, \quad (4.3)$$

is nearly constant because neither U_e nor U_0 changes appreciably with $\dot{m}_{\text{air}}$. For the same reason, the specific fuel consumption (SFC) also remains flat: both F_{net} and $\dot{m}_f$ scale with $\dot{m}_{\text{air}}$, so their ratio is invariant.

Taken together, these results show that, within this 0D framework, varying the air mass flow rate simply scales the "size" of the engine. Increasing $\dot{m}_{\text{air}}$ produces a larger version of the same cycle with higher absolute thrust and fuel flow, but with unchanged specific thrust and SFC, meaning the underlying cycle efficiency is unaffected.

4.3 Varying Compressor Ratio

While the results indicate a monotonic improvement in SFC up to $\pi_c = 15$, practical engine design faces limits not captured in this 0-D analysis. First, the assumption of constant polytropic efficiency ($\eta_{\text{poly}} = 0.83$) is optimistic; in reality, efficiency often degrades at higher pressure ratios due to reduced blade heights and increased leakage losses in the rear compressor stages. Second, the continuous drop in Specific Thrust implies that a higher-CPR engine would be physically larger and heavier to produce the same net thrust, potentially negating the fuel burn benefits at the aircraft level. Modern engines mitigate this by increasing the

Overall Pressure Ratio (OPR) to 40+ while simultaneously increasing the Bypass Ratio, a coupling that allows for both high thermal efficiency (from OPR) and high propulsive efficiency (from BPR), which was held constant in this specific study.

4.4 Component Sensitivity: Why The Turbine Dominates

The sensitivity study makes the component hierarchy unambiguous. Varying the turbine polytropic efficiency produces the steepest gradients in both thrust and SFC, the compressor shows strong leverage on SFC but almost no leverage on thrust, and the fan has the least impact. This ranking reflects the way exergy is distributed in the cycle.

The turbine is the only component that converts chemical potential into shaft work. For a fixed compressor work requirement, any drop in $\eta_{p,T}$ forces a larger enthalpy drop across the turbine to produce the same shaft power. Extra Δh_t comes at the expense of a deeper pressure drop from $P_{0,5}$ to $P_{0,6}$, which directly erodes the total-pressure budget available to the nozzle. The result is a "double penalty": more fuel is required to hit the same $T_{0,5}$, and less total pressure survives to be turned into jet kinetic energy. Improving $\eta_{p,T}$ reverses this process; the turbine does the same shaft work while leaving more pressure standing at Station 6, which translates almost one-for-one into higher exhaust momentum and lower SFC.

The compressor, in contrast, controls thermal efficiency but only weakly affects thrust at fixed $T_{0,5}$. A more efficient compressor reduces the work needed to achieve the same pressure ratio, so the turbine can extract that work with a smaller enthalpy drop. Fuel flow falls because the shaft system needs less power, and SFC improves. However, the nozzle inlet total temperature and pressure change only modestly, so net thrust barely moves. The fan sits at lower pressure ratio and temperature than the core, so its efficiency changes redistribute power and SFC in the expected directions but with significantly smaller leverage than the core components.

From a design perspective, these trends are very informative. If money is to be spent on improving component efficiency, it should go into the turbine first, the core compressor second, and the fan last.

4.5 Compressor Pressure Ratio: Thermal Efficiency versus Specific Thrust

The compressor-pressure-ratio sweep from $\pi_c = 4$ to $\pi_c = 15$ at fixed $T_{0,5}$ illustrates the classic Brayton trade-off between thermal efficiency and specific work output. As π_c increases, SFC falls monotonically, as predicted by textbook relations for ideal-cycle thermal efficiency. The

engine burns less fuel per unit thrust because more of the heat addition occurs at a higher average temperature.

The cost is incurred in terms of specific thrust. Raising π_c increases $T_{0,4}$ and therefore the compressor work. With $T_{0,5}$ fixed, the turbine must expand further to supply this extra work, which drives $T_{0,6}$ and $P_{0,6}$ downward and leaves less enthalpy for the nozzle. Exit velocity falls, and both net thrust and specific thrust slide downward with increasing π_c .

The constant-thrust optimization study makes this tension clearer. For a fixed $F_{\text{net}} \approx 67$ kN, the SFC continues to decrease all the way to the project limit of $\pi_c = 15$, and an extended sweep out to $\pi_c = 40$ reveals a true thermodynamic minimum around $\pi_c \approx 18.7$. Beyond this point, the additional compression work cannot be recovered efficiently, and the required mass flow and engine frontal area begin to grow. In reality, the optimum would shift even lower once stage efficiency degradation, tip-clearance losses and increased engine weight are included. This is consistent with the F100's overall pressure ratio of 24:1 and low bypass ratio, which bias the design toward high thrust density rather than minimum SFC as this engine wasn't designed to be particularly efficient.

4.6 Turbine Inlet Temperature

The turbine-inlet-temperature sweep is where the mixer model starts to matter. As $T_{0,5}$ is raised from 1200 to 1850 K, net thrust climbs monotonically, as expected: hotter core gas means higher nozzle exit velocity and more momentum flux. The SFC, however, does not simply rise with $T_{0,5}$. Instead, the SFC curve flattens and develops a clear minimum in the 1400–1450 K range, as shown in Figure 3.9. The absolute minimum occurs at $T_{0,5} \approx 1420.4$ K, where the model predicts $F_{\text{net}} \approx 50.4$ kN; this point is marked explicitly on the SFC and thrust curves in Figure 3.9 and defines the most efficient continuous operating condition of the cycle/engine.

This behaviour is a direct consequence of enforcing a 1-D momentum balance in the mixer instead of treating it as a simple enthalpy balance at ⑧. At low TIT, the turbine must expand aggressively to supply compressor and fan work, which drives $P_{0,6}$ well below the bypass total pressure $P_{0,7}$. The bypass stream then has to diffuse or “waste” part of its pressure to match the core static pressure at the mixing plane. The resulting shear layer and pressure mismatch generate entropy and effectively throw away available work, which shows up as elevated SFC.

As $T_{0,5}$ increases toward 1420 K, the turbine expansion ratio required to meet the shaft-power demand falls, and $P_{0,6}$ rises toward $P_{0,7}$. The two streams enter the mixer with nearly matched total and static pressures, so the momentum deficit and associated mixing losses

are minimised. That point is the “mixing minimum” observed in the SFC curve. Pushing $T_{0,5}$ beyond this region continues to increase thrust, but now the penalty is dominated by propulsive efficiency: the jet Mach number climbs, the exhaust kinetic energy per unit thrust grows faster than the useful thrust work, and SFC rises again.

Crucially, this numerically identified minimum in the SFC when varying TIT is not just a random dip in the SFC. Table 1.1 lists the published *maximum continuous* dry rating of the F100–PW–100 as approximately 55 kN, only about 9% above the model prediction of 50.4 kN at $T_{0,5} = 1420.4$ K. In other words, by scanning Figure 3.9 for the SFC minimum and reading off the corresponding thrust, the simulation independently recovers an operating point that sits essentially on top of the real engine’s continuous-thrust rating. That agreement was not imposed as an input: it emerges from the coupled turbine, mixer and nozzle physics in the solver. This provides a strong, additional validation that the momentum-based mixing model and enthalpy-corrected property treatment are capturing the correct thermodynamic behaviour of the F100 cycle, and that the real engine is indeed sized to cruise near this TIT condition, using higher- $T_{0,5}$ “military power” only when extra thrust is worth the fuel penalty.

It is crucial to distinguish this operating point from the “Military Power” rating (≈ 65 kN dry), which operates at a higher TIT (likely closer to the 1680 K used in Task 1). The simulation confirms that pushing from 1400 K to 1680 K yields the extra ≈ 10 kN of thrust required for combat maneuvers, but at the cost of moving away from the SFC optimum, a classic trade-off in fighter engine design.

4.7 Afterburner Thermodynamics: Poor Exergy Usage

The afterburner results are exactly as unforgiving as thermodynamics predicts. Lighting the reheat system boosts net thrust from 67.5 to 100.5 kN, roughly a 50 % increase, and raises specific thrust to almost 1000 N·s/kg. In exchange, the total fuel flow climbs from 1.33 to 4.33 kg/s and the SFC more than doubles.

From a cycle perspective, the main burner adds heat at the high pressure of the compressor discharge, where the added enthalpy can be converted efficiently into shaft work and then into jet kinetic energy. The afterburner, by contrast, adds heat at the much lower pressure level downstream of the turbine. In Rayleigh-line terms, the flow entering the afterburner is already high-velocity and low-pressure; injecting fuel and burning at this point pushes the state almost vertically up the Rayleigh line, generating a large entropy increase for a relatively modest gain in stagnation pressure. The nozzle can and does convert the higher $T_{0,9}$ into a faster jet, but the exergy of that added heat is intrinsically poor compared to heat

added in the core.

The underprediction of wet thrust relative to the published 106.4 kN rating is consistent with this simplified picture. The present model does not include variable nozzle throat area, detailed flameholder losses or turbine cooling flows, and it treats the afterburner as a single, spatially uniform control volume. Any of these effects could shift the real engine slightly closer to the ideal Rayleigh line and recover a few percent more thrust. At the level of fidelity used here, we can conclude that afterburning is an effective but thermodynamically expensive way to increase thrust.

4.8 Model limitations

The agreement with F100 published performance data [3, 4], is encouraging, but the 0D model has hard limits regarding fidelity that matter in the development of actual engine models as discussed in [7] and further elaborated by NASA and other studies [9, 16]:

- **Property model:** The cp polynomials are extrapolated well beyond their fitted range, which is acceptable for the present project but not for engines operating deeper into the $T > 2000$ K regime.
- **Missing secondary flows:** Turbine blade and vane cooling, bleed flows for stability, and tip-clearance losses are all lumped into effective polytropic efficiencies and pressure-loss factors. This is the main reason a clean station model can still be a few percent off in thrust and SFC.
- **Mixer idealisation:** Even with the 1-D momentum model, the mixer is treated as a single cell with a prescribed loss factor. Real mixed-flow engines exploit tailored area distributions and three-dimensional mixing devices to control pressure recovery and suppress separation.
- **Geometry and off-design behaviour:** Variable stators, variable nozzles and corrected-speed scheduling are frozen out of the present analysis. As a result, off-design trends with flight Mach number, altitude and spool speed cannot be trusted without a more sophisticated matching framework.
- **Chemistry and emissions:** Combustion is represented solely through enthalpy addition. No mention of chemical kinetics and/or the formation of NO_x , CO or unburned hydrocarbons is part of the present analysis; a Cantera-based extension with multi-species chemistry would be required to capture the very important role of chemical kinetics in combustion.

Despite these limitations, the 0D cycle model is a great base level model: it captures the dominant thermodynamic building blocks of the F100–PW–100, reproduces published thrust

levels to great accuracy, and exposes where the engine is trading fuel, temperature, to fulfill the design requirements.

5 Conclusion

This study implemented and validated a zero-dimensional thermodynamic cycle model of the Pratt & Whitney F100-PW-100 turbofan engine. By incorporating temperature-dependent specific heats and an enthalpy-based iterative solver, the analysis captured the non-linear behavior of the Brayton cycle across a wide range of operating conditions. The baseline simulation reproduced the engine’s sea-level static performance (SLS) with good fidelity, predicting a dry military power thrust of 67.52 kN and a maximum afterburning thrust of 100.53 kN. These values deviate by only +3.2% and −5.7% from published data, respectively, which indicates that a station-averaged 0D approach is adequate for first-order performance characterization when caloric imperfections are considered in detail.

The parametric sensitivity analysis revealed a clear thermodynamic hierarchy within the engine architecture. The high-pressure turbine emerged as the critical element for cycle efficiency, since its polytropic efficiency exerts the strongest influence on both net thrust and specific fuel consumption (SFC). Turbine inefficiencies impose a double penalty on the cycle: they require a larger pressure drop to satisfy the compressor work demand, which increases fuel consumption while reducing the stagnation pressure available for thrust generation. In contrast, fan and compressor efficiencies primarily affect thermal efficiency, with a secondary impact on propulsive force.

A key outcome of this work was the identification of a ”mixing minimum” in SFC. The cycle solver predicted a local SFC minimum at a turbine inlet temperature of approximately 1420 K, where the net thrust (50.4 kN) is close to the F100’s published *maximum continuous thrust rating*. This behavior is consistent with the hypothesis that the continuous operating point lies near a condition where core and bypass pressures are closely matched, which reduces entropy generation in the mixer. In contrast, operation at Military Power, (*absolute maximum thrust without afterburner*) at 1680 K, pushes the engine away from this thermodynamic optimum and accepts a deliberate penalty in propulsive efficiency in exchange for higher thrust for combat maneuvering.

The investigation into thrust augmentation quantified the high thermodynamic cost of afterburning. The reheat system increased specific thrust by nearly 50%, but did so by approximately tripling the fuel flow rate. This result illustrates the poor fuel economy associated with Rayleigh-line heat addition at relatively low pressure, where large increases in entropy produce diminishing returns in effective stagnation pressure and exit velocity.

The compressor pressure ratio study indicated a theoretical thermal efficiency maximum near $\pi_c \approx 18.7$, which suggests that the F100's design pressure ratio of 24.1 sacrifices some thermal efficiency in order to gain higher specific thrust and reduced frontal area compared to high-bypass transport engines.

The present 0D model provides robust trend predictions, but its fidelity is limited by the exclusion of turbine cooling bleed flows and detailed chemical kinetics. The omission of bleed air is a plausible source of the remaining thrust overprediction in dry mode, since real engines typically divert 2.5–10% of compressor air for cooling and sealing. Future work will address these gaps by coupling the cycle solver to a Cantera framework using detailed Jet A-1 chemistry incorporating surrogate chemical kinetic models. This extension is intended to support the prediction of minor species such as NO_x and CO and to incorporate more realistic hot-gas properties. In this way, the methodology can be extended from purely thermodynamic performance assessment toward preliminary combustor design and efficiency.

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Appendices

A Additional Figures and Tables

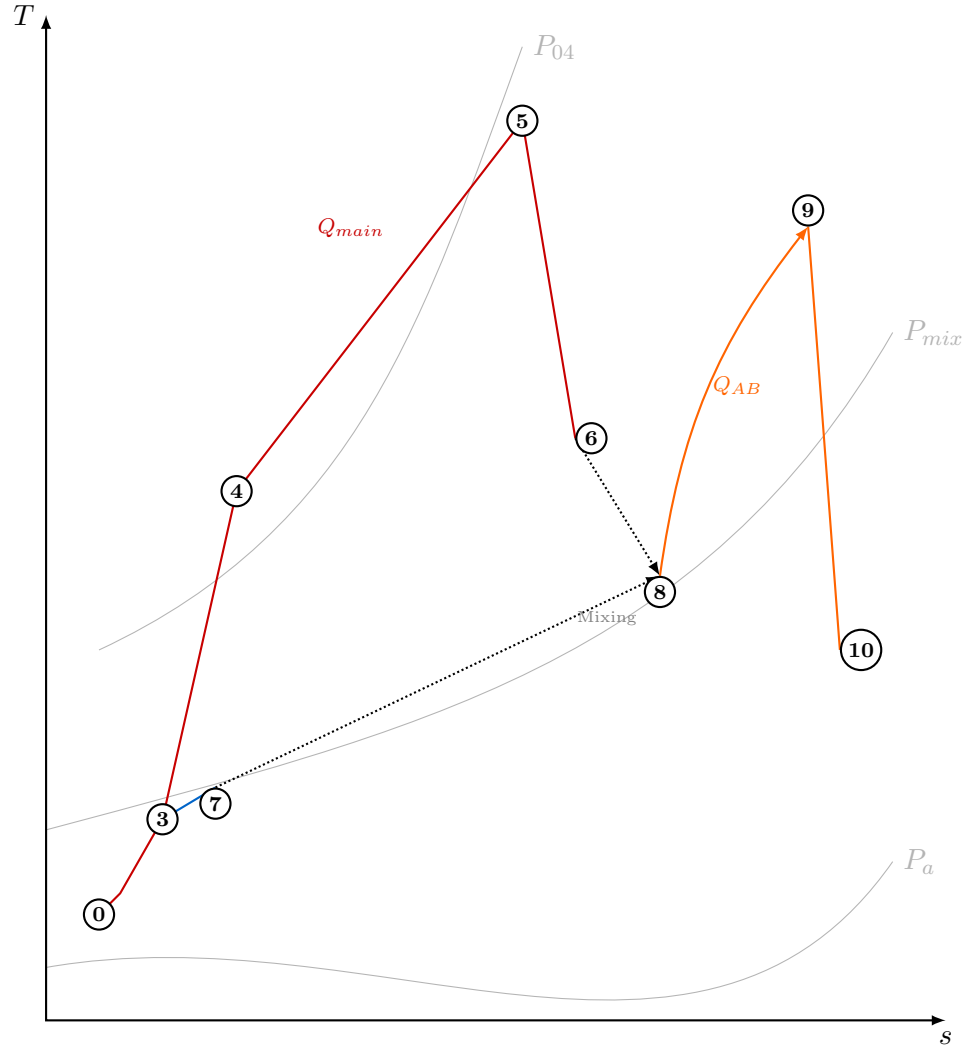


Figure A.1: Qualitative Temperature-Entropy ($T-s$) diagram illustrating the thermodynamic cycle of the F100-PW-100 mixed-flow turbfan engine in augmented (afterburning) mode. The red and blue paths trace the core and bypass streams, respectively. Key features include the main combustion ($4 \rightarrow 5$), the irreversible constant-pressure mixing of the two streams (converging at Station 8), and the reheat process (Q_{AB}) preceding final nozzle expansion. Isobaric divergence is exaggerated to enhance the visibility of the mixing and afterburning distinct phases.

B Supplemental Methods

B.1 On-design Turbofan Cycle Calculations

Presented below are the methodology and assumptions used for hand calculations.

Given parameters

Ambient conditions and gas properties:

$$\begin{aligned}
 P_a &= 90.31 \times 10^3 \text{ Pa}, & T_a &= 288.15 \text{ K}, & R &= 287 \text{ J kg}^{-1}\text{K}^{-1}, \\
 \gamma_{\text{cold}} &= 1.4, & c_{p,\text{cold}} &= 1005 \text{ J kg}^{-1}\text{K}^{-1}, \\
 \gamma_{\text{hot}} &= 1.333, & c_{p,\text{hot}} &= 1150 \text{ J kg}^{-1}\text{K}^{-1}.
 \end{aligned}$$

Engine parameters:

$$\begin{aligned}
 \dot{m}_{\text{air}} &= 101.6 \text{ kg s}^{-1}, & \text{BPR} &= 0.69, \\
 \text{PR}_{\text{intake}} &= 0.99, & \text{PR}_{\text{fan}} &= 3.0, \\
 \eta_{\text{poly,fan}} &= 0.79, \\
 \text{PR}_{\text{comp,duct}} &= 0.99, & \text{PR}_c &= 8.04, & \eta_{\text{poly,comp}} &= 0.83, \\
 Q &= 43323 \times 10^3 \text{ J kg}^{-1}, & \eta_{\text{burn}} &= 0.98, & \text{PR}_{\text{burn}} &= 0.96, \\
 T_{\text{IT}} &= 1680 \text{ K}, \\
 \eta_{\text{poly,turb}} &= 0.86, & \eta_{\text{mech}} &= 0.99, \\
 \text{PR}_{\text{byp,duct}} &= 0.93, & \text{PR}_{\text{mixer}} &= 0.99, \\
 T_{\text{aft}} &= 2035.4 \text{ K}, \\
 \eta_{\text{nozz}} &= 0.99, & A_{\text{exit}} &= 0.726 \text{ m}^2.
 \end{aligned}$$

Mass flow split between core and bypass:

$$\begin{aligned}
 \dot{m}_{\text{core,air}} &= \frac{\dot{m}_{\text{air}}}{1 + \text{BPR}}, \\
 \dot{m}_{\text{byp}} &= \dot{m}_{\text{air}} - \dot{m}_{\text{core,air}}, \\
 \dot{m}_{\text{tot}} &= \dot{m}_{\text{core,air}} + \dot{m}_{\text{byp}}.
 \end{aligned}$$

Intake and fan

Apply intake pressure ratio and take $T_{02} = T_a$:

$$P_{02} = P_a \text{PR}_{\text{intake}},$$

$$T_{02} = T_a.$$

Apply fan pressure ratio with polytropic relation:

$$P_{03} = \text{PR}_{\text{fan}} P_{02},$$

$$T_{03} = T_{02} \left(\frac{P_{03}}{P_{02}} \right)^{\frac{\gamma_{\text{cold}} - 1}{\gamma_{\text{cold}} \eta_{\text{poly, fan}}}}.$$

High pressure compressor

Include core duct loss then compressor pressure ratio:

$$P_{04} = \text{PR}_{\text{comp, duct}} \text{PR}_c P_{03}.$$

Polytropic temperature rise across the compressor:

$$T_{04} = T_{03} \left(\frac{P_{04}}{P_{03}} \right)^{\frac{\gamma_{\text{cold}} - 1}{\gamma_{\text{cold}} \eta_{\text{poly, comp}}}}.$$

Combustor and fuel flow

Set turbine inlet temperature and use an energy balance with burner efficiency to get fuel flow:

$$T_{05} = T_{\text{IT}},$$

$$\dot{m}_{\text{fuel}} = \frac{\dot{m}_{\text{core, air}}}{\frac{Q \eta_{\text{burn}}}{c_{p, \text{hot}}(T_{05} - T_{04})} - 1}.$$

Total core mass into the turbine:

$$\dot{m}_{\text{core}} = \dot{m}_{\text{core, air}} + \dot{m}_{\text{fuel}}.$$

Turbine power balance

Compute compressor plus fan power that the turbine must supply:

$$\dot{W}_{\text{turb,req}} = \dot{m}_{\text{air}} c_{p,\text{cold}} (T_{03} - T_{02}) + \dot{m}_{\text{core,air}} c_{p,\text{cold}} (T_{04} - T_{03}).$$

Use this power balance to find T_{06} :

$$T_{06} = T_{05} - \frac{\dot{W}_{\text{turb,req}} \eta_{\text{mech}}}{\dot{m}_{\text{core}} c_{p,\text{hot}}}.$$

Apply burner pressure ratio then polytropic expansion across the turbine:

$$P_{05} = P_{04} \text{PR}_{\text{burn}},$$

$$P_{06} = P_{05} \left(\frac{T_{06}}{T_{05}} \right)^{\frac{\gamma_{\text{hot}}}{(\gamma_{\text{hot}} - 1) \eta_{\text{poly,turb}}}}.$$

Bypass duct and mixer

Bypass stream pressure loss:

$$P_{07} = P_{03} \text{PR}_{\text{byp,duct}},$$

$$T_{07} = T_{03}.$$

Mass weighted mixing in pressure and temperature, then mixer loss:

$$P_{08} = \left(\frac{\dot{m}_{\text{core}} c_{p,\text{hot}} P_{06} + \dot{m}_{\text{byp}} c_{p,\text{cold}} P_{07}}{\dot{m}_{\text{tot}} c_{p,\text{hot}}} \right) \text{PR}_{\text{mixer}},$$

$$T_{08} = \frac{\dot{m}_{\text{core,air}} c_{p,\text{hot}} T_{06} + \dot{m}_{\text{byp}} c_{p,\text{cold}} T_{07}}{\dot{m}_{\text{tot}} c_{p,\text{hot}}}.$$

Afterburner off

With no afterburner:

$$T_{09} = T_{08},$$

$$P_{09} = P_{08}.$$

Nozzle and thrust

Compute the critical pressure ratio to check for choking:

$$\text{PR}_{\text{crit}} = \left(\frac{\gamma_{\text{hot}} + 1}{2} \right)^{\frac{\gamma_{\text{hot}}}{\gamma_{\text{hot}} - 1}}.$$

Exit pressure and ideal exit temperature:

$$\text{if } \frac{P_{09}}{P_a} > \text{PR}_{\text{crit}} \quad (\text{choked})$$

$$P_e = \frac{P_{09}}{\text{PR}_{\text{crit}}},$$

$$T_{e,\text{ideal}} = \frac{2}{\gamma_{\text{hot}} + 1} T_{09},$$

$$\text{else} \quad (\text{unchoked})$$

$$P_e = P_a, \quad T_{e,\text{ideal}} = T_{09} \left(\frac{P_a}{P_{09}} \right)^{\frac{\gamma_{\text{hot}} - 1}{\gamma_{\text{hot}}}}.$$

Account for nozzle efficiency and get exit velocity:

$$T_e = T_{09} - \eta_{\text{nozz}} (T_{09} - T_{e,\text{ideal}}),$$

$$v_e = \sqrt{2 c_{p,\text{hot}} (T_{09} - T_e)}.$$

Net thrust from momentum plus pressure term:

$$F = \dot{m}_{\text{tot}} v_e + (P_e - P_a) A_{\text{exit}} \approx 84,000 \text{N}.$$

Validation Discrepancy The manual calculations presented above rely on constant thermodynamic properties ($\gamma_{\text{cold}} = 1.4$, $\gamma_{\text{hot}} = 1.333$) for analytical tractability, whereas the primary computational model incorporates temperature-dependent specific heats ($c_p(T)$). Since the constant-property assumption fails to capture the increased heat capacity of the working fluid at high temperatures, it under-predicts the specific work required by the core components. Consequently, the results obtained via hand calculation are expected to slightly overestimate the engine's net performance metrics relative to the higher-fidelity numerical solution.

B.2 Supplementary Python Implementations for Turbofan Cycle Analysis

In addition to the primary MATLAB based analysis used for the assessed portion of this project, a suite of Python scripts was developed to explore alternative modelling assumptions and to build a reusable turbofan analysis toolchain. These implementations are still under active development and will be released in a public GitHub repository after the course submission. The scripts are listed here for completeness and to document the modelling philosophy that informed the final results.

Overview of Implementations

`turbofan_static.py`. Baseline Python script created as a near one to one translation of the original MATLAB code. The script follows a station by station cycle analysis (intake, fan, compressor, combustor, turbine, mixer, nozzle) with temperature dependent specific heats evaluated from polynomial correlations. Each major component uses an iterative loop that updates c_p and γ until the temperature field converges.

`turbofan_static_2_sim.py`. Refined baseline with improved input handling, mass flow checks, and a more explicit control volume formulation for the nozzle. Total mass flow at the nozzle exit is compared against both the imposed engine mass flow and the geometric choking condition, which is useful for diagnosing inconsistent operating points.

`turbofan_static_3_sim.py`. Version that introduces switchable thermodynamic property models. A simple factory function selects between polynomial fits and CoolProp based properties. This allows direct comparison between hand calculation style properties and more realistic real gas behaviour, especially in the colder parts of the cycle.

`turbofan_static_4_sim.py`. Extension of the previous script that adds optional Cantera based hot gas properties while retaining CoolProp for the cold stream. Separate functions are used for $c_{p,cold}(T)$ and $c_{p,hot}(T)$, which makes it possible to treat the combustor products with equilibrium chemistry while keeping the rest of the cycle relatively simple.

`turbofan_static_5_sim.py`. Most comprehensive implementation at this stage. This script combines CoolProp, Cantera equilibrium chemistry, optional humid air corrections, hooks for bleed and turbine cooling flows, and both frozen and equilibrium nozzle models. For the main combustor, the code can read the Stanford HyChem Jet A-1 mechanism from

a YAML file and use it to supply detailed chemical kinetics for the reacting core flow. A surrogate model is being developed to constrain the set of candidate reaction pathways by assigning probabilities to different reaction types (first order, second order, dissociation and recombination reactions) and retaining only the most likely mechanisms for a given operating condition. This probabilistic reduction of the full HyChem mechanism is still a work in progress and has proven more challenging than originally expected, so at present it is used only in exploratory runs and not for the baseline results in this report.

turbofan_static_6_sim.py. Streamlined version tailored for batch studies and future parametric work. It uses a hierarchical property fallback (CoolProp, then Pyromat, then polynomials), simplifies the mixer logic, and writes all operating point results to CSV files in a dedicated results directory to support plotting and data post processing. The script retains the interface to the Stanford HyChem Jet A-1 YAML mechanism and the associated surrogate reaction model from **turbofan_static_5_sim.py**, so that detailed combustor chemistry can be toggled on for future studies. For the current project, these advanced kinetics options are disabled in the main parametric sweeps to keep the cycle model consistent and numerically robust, but the infrastructure is in place for later, more chemistry focused work.

Modelling Assumptions and Sensitivity

Across all versions the core methodology is consistent. Polytropic relations are used for compressors and turbines, with total to total temperature changes obtained from the usual efficiency conversion. Mass and energy conservation are enforced at each control volume, and the turbine power balance is solved by iterating on T_{06} until the shaft work matches the sum of fan and compressor power, including mechanical losses. The mixer combines the core and bypass streams using a coupled momentum and energy balance, either with a simple algebraic closure or with a relaxed iteration to improve numerical stability.

The different scripts mainly explore how predicted thrust and specific fuel consumption respond to:

- choice of property model (polynomial fits versus CoolProp and Cantera, with optional HyChem based chemistry in the combustor),
- treatment of mixer pressure loss and mixing closure,
- details of the nozzle model and mass flow consistency checks,
- inclusion or exclusion of secondary effects such as bleeds, cooling flows, afterburner operation, and detailed chemical kinetics.

The spread in predicted thrust between scripts reflects these different physical and

numerical assumptions. Rather than forcing agreement to a single number, the approach was to understand which assumptions drive the spread and then select a conservative, physically defensible model for the final parametric studies.

Path Forward

For the formal project results all figures and tables are generated from a single, consolidated Python script derived from `turbofan_static_6_sim.py`. This script is structured for batch execution, accepts engine and cycle parameters as inputs, and outputs thrust, specific thrust, and specific fuel consumption for parametric sweeps. The broader code base described in this appendix will continue to be developed beyond the course and will be made publicly available in a version controlled GitHub repository, along with documentation and example cases. The HyChem based surrogate kinetics framework will be completed and integrated into that public release once the numerical issues have been resolved. It is very likely that I will have to rewrite most of this code in C++ for efficiency; the current Python scripts are not computer-friendly at the moment.